John M. Holte

Fractal dimension of arithmetical structures of generalized binomial coefficients modulo a prime,

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Abstract

Given a sequence (u_n) of positive integers generated by $u_1 = 1, u_2 = a, u_n = au_{n-1} + bu_{n-2} (n \geq 3)$, define the generalized factorial by $[n]! = u_1 u_2 \cdots u_n$ and the generalized binomial coefficient by $C(i,j) = [i+j]!/([i]![j]!)$. Assume that the prime p does not divide b. Let $r = \min\{n : p | u_n\}$. **Theorem 1 (Asymptotic abundance of residues):** $\#\{(i,j) | 0 \leq i,j < rp^k \text{ and } C(i,j) \equiv \rho (\mathrm{mod}\ p)\} \sim \frac{r(r+1)}{2(p-1)} \binom{p+1}{2}^k$ as $k \to \infty$ for $\rho = 1, \ldots, p-1$. **Theorem 2 (Fractal dimension):** Let $s_k = rp^k$. The Hausdorff dimension of $\bigcap_k \bigcup_{i,j<s_k} \{[i/s_k, (i+1)/s_k) \times [j/s_k, (j+1)/s_k) : p \not| C(i,j)\}$ is $\log \binom{p+1}{2} / \log p$.