Ming Wu and Hao Pan
Sums of products of Bernoulli numbers of the second kind,
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Abstract

The Bernoulli numbers of the second kind b_n are defined by

$$\sum_{n=0}^{\infty} b_n t^n = \frac{t}{\log(1+t)}.$$

In this paper, we give an explicit formula for the sum

$$\sum_{\substack{j_1+j_2+\cdots+j_N=n \\ j_1,j_2,\ldots,j_N\geq 0}} b_{j_1} b_{j_2} \cdots b_{j_N}.$$

We also establish a q-analogue for

$$\sum_{k=0}^{n} b_k b_{n-k} = -(n-1)b_n - (n-2)b_{n-1}.$$