ERRATA FOR SOME IDENTITIES INVOLVING THE POWERS OF THE GENERALIZED FIBONACCI NUMBERS

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On page 9, (10) should be

$$E^2(x) = \frac{(V_3 - 2q^3x)E_1'(x) - 4q^3E_1(x)}{(p^2-4q)^2U_3} - \frac{3[(p-2q^2x)E_2'(x) - 4q^2E_2(x)]}{(p^2-4q)^2}$$
$$- \frac{6q}{p(p^2-4q)^{\frac{7}{2}}}\left[\frac{\alpha^6}{1-\alpha^3x} - \frac{\alpha^2q(V_2+q)}{1-\alpha qx} + \frac{\beta^2q(V_2+q)}{1-\beta qx} - \frac{\beta^6}{1-\beta^3x}\right].$$

On page 10, (11-15) should be, respectively,

$$E^3(x) = \frac{[(V_3-2q^3x)^2E_1''(x) - 14q^3(V_3-2q^3x)E_1'(x) + 32q^6E_1(x)]}{2(p^2-4q)^4U_3^2}$$
$$+ \frac{9[(p-2q^2x)^2E_2''(x) - 14q^3(p-2q^2x)E_2'(x) + 32q^4E_2(x)]}{2(p^2-4q)^4}$$
$$- \frac{9q}{p(p^2-4q)^5}\left\{\frac{\alpha^9}{(1-\alpha^3x)^2} - \left[\frac{\alpha^6q(V_2+q)}{\alpha-\beta} + \frac{\alpha^3q^3V_3}{\alpha^3-\beta^3}\right]\frac{1}{1-\alpha^3x}\right.$$
$$+ \left[\frac{\beta^3q^3V_3}{\alpha^3-\beta^3} + \frac{\beta^6q(V_2+q)}{\alpha-\beta}\right]\frac{1}{1-\beta^3x} + \frac{\alpha^4q^2(V_2+q)}{(\alpha-\beta)(1-\alpha qx)}$$
$$\left. - \frac{\beta^4q^2(V_2+q)}{(\alpha-\beta)(1-\beta qx)} + \frac{\beta^9}{(1-\beta^3x)^2}\right\}$$

$$+ \frac{27q^2}{p(p^2-4q)^5}\left\{\frac{\alpha^9}{(\alpha^2-\beta^2)(1-\alpha^3x)} + \left[\frac{\alpha V_1q^2(V_2+q) - \alpha^6q}{\alpha-\beta}\right.\right.$$
$$\left. - \frac{\beta^3q^3}{\alpha^2-\beta^2}\right]\frac{1}{1-\alpha qx} - \frac{\alpha^3q(V_2+q)}{(1-\alpha qx)^2} + \left[\frac{\beta^6q - \beta V_1q^2(V_2+q)}{\alpha-\beta}\right.$$
$$\left.\left. + \frac{\alpha^3q^3}{\alpha^2-\beta^2}\right]\frac{1}{1-\beta qx} - \frac{\beta^3q(V_2+q)}{(1-\beta qx)^2} - \frac{\beta^9}{(\alpha^2-\beta^2)(1-\beta^3x)}\right\}$$

$$\sum_{a+b=n} U_a^2U_b^2 = \frac{[-2nqU_{n-1} + p(n-1)U_n]V_n}{(p^2-4q)^2} - \frac{2q}{(p^2-4q)^2}\left(\frac{U_{2n}}{U_2} - nq^{n-1}\right), \quad n \geq 1,$$

$$\sum_{a+b+c=n} U_a^2 U_b^2 U_c^2 = \frac{U_n}{2(p^2-4q)^3}\Big[p^2(n-2)(n-1)U_n - 2pq(n-2)(2n+1)U_{n-1} + 4q^2(n-1)(n+1)U_{n-2}\Big] - \frac{3q}{(p^2-4q)^3}\left[\frac{(n-2)U_{2n}}{U_2} + \frac{4q(q^{n-2}V_3 - V_{2n-1})}{p(p^2-4q)} + (n+2)(n-2)q^{n-1}\right], \quad n \geq 2.$$

$$\sum_{a+b=n} U_{ak}^2 U_{bk}^2 = \frac{[-2nq^k U_{nk-k} + (n-1)V_k U_{nk}]V_{nk}}{(p^2-4q)^2 U_k} - \frac{2q^k}{(p^2-4q)^2}\left(\frac{U_{2nk}}{U_{2k}} - nq^{nk-k}\right), \quad n \geq 1,$$

$$\sum_{a+b+c=n} U_{ak}^2 U_{bk}^2 U_{ck}^2 = \frac{U_{nk}}{2(p^2-4q)^3 U_k^2}\Big[V_k^2(n-2)(n-1)U_{nk} - 2V_k q^k(n-2)(2n+1)U_{nk-k} + 4q^{2k}(n-1)(n+1)U_{nk-2k}\Big] - \frac{3q^k}{(p^2-4q)^3}\left[\frac{(n-2)U_{nk}}{U_{2k}} + \frac{4q^k(q^{nk-2k}V_{3k} - V_{2nk-k})}{V_k U_k^2(p^2-4q)} + (n+2)(n-2)q^{nk-k}\right], \quad n \geq 2.$$

On pages 11-12, Theorem 2 and Corollary 2 should be, respectively,

Theorem 2: Let $\{U_n\}$ be the generalized Fibonacci sequence. Then

$$\sum_{a+b=n} U_a^3 U_b^3 = \frac{(n-1)V_3 U_{3n} - 2nq^3 U_{3n-3}}{(p^2-4q)^3 U_3} - \frac{9q^n[2nqU_{n-1} - p(n-1)U_n]}{(p^2-4q)^3} - \frac{6q[U_{3n} - (V_2+q)q^{n-1}U_n]}{p(p^2-4q)^3}, \tag{17}$$

$$\sum_{a+b+c=n} U_a^3 U_b^3 U_c^3 = \frac{1}{2(p^2-4q)^5 U_3^2}\Big[(n^2-3n+2)V_3^2 U_{3n} - 4q^3(n^2-5n+6)V_3 U_{3n} + 4q^6(n^2-7n+12)U_{3n-6}\Big] - \frac{7q^3[(n-2)V_3 U_{3n-3} - 2(n-3)q^3 U_{3n-6}]}{(p^2-4q)^5 U_3^2} + \frac{16q^6 U_{3n-6}}{(p^2-4q)^5 U_3^2} - \frac{27}{2(p^2-4q)^5}\Big[(n^2-3n+2)p^2 q^n U_n - 4(n^2-5n+6)q^{n+1}U_{n-1} + 4(n^2-7n+12)q^4 U_{3n-6}\Big] + \frac{189q^{n+2}[(n-2)pU_{n-1} - 2(n-3)qU_{n-2}] - 48q^{n+2}U_{n-2}}{(p^2-4q)^5}$$

$$- \frac{9q}{p(p^2-4q)^5} \Big[(n-2)V_{3n} - (V_2+q)qU_{3n-3} - \frac{q^3 V_3 U_{3n-6}}{U_3} + (V_2+q)q^{n-1}U_{n+1} \Big] + \frac{27q^2}{p(p^2-4q)^5} \Big[\frac{U_{3n}}{U_2} + (V_2+q)V_1 q^{n-1} U_{n-2} - q^{n-2}U_{n+3} - \frac{q^{n+3}U_{n-6}}{U_2} - (n-2)(V_2+q)q^{n-2}V_n \Big].$$

Corollary 2: Let $\{U_n\}$ be the generalized Fibonacci sequence and k be a positive integer. Then

$$\sum_{a+b=n} U_{ak}^3 U_{bk}^3 = \frac{(n-1)V_{3k}U_{3kn} - 2nq^{3k}U_{3k(n-1)}}{U_{3k}(p^2-4q)^3} - \frac{9q^{kn}[2nq^k U_{nk-k} - V_k(n-1)U_{kn}]}{U_k(p^2-4q)^3} - \frac{6q^k[U_{3kn} - (V_{2k}q^{kn-k} + q^{kn})U_{nk}]}{U_k V_k (p^2-4q)^3}$$

$$\sum_{a+b+c=n} U_{ak}^3 U_{bk}^3 U_{ck}^3 = \frac{1}{2U_{3k}^2(p^2-4q)^5} \Big[(n^2-3n+2)V_{3k}^2 U_{3kn} - 4(n^2-5n+6)q^{3k}V_{3k}U_{3nk} + 4(n^2-7n+12)q^{6k}U_{3nk-6k} \Big] - \frac{7q^{3k}[(n-2)V_{3k}U_{3nk-3k} - 2(n-3)q^{3k}U_{3nk-6k}]}{U_{3k}^2(p^2-4q)^5} + \frac{16q^{6k}U_{3nk-6k}}{U_{3k}^2(p^2-4q)^5} - \frac{27}{2U_k^2 V_k (p^2-4q)^5} \Big[(n^2-3n+2)V_k^2 q^{nk} U_{nk} - 4(n^2-5n+6)q^{nk+k}U_{nk-k} + 4(n^2-7n+12)q^{4k}U_{3nk-6k} \Big] + \frac{q^{nk+2k}\{189[(n-2)V_k U_{nk-k} - 2(n-3)q^k U_{nk-2k}] - 48U_{nk-2k}\}}{U_k^2(p^2-4q)^5} - \frac{9q^k}{V_k U_k^2 (p^2-4q)^5} \Big[(n-2)U_k V_{3nk} - (V_{2k}+q^k)q^k U_{3nk-3k} - q^{3k}U_k V_{3k}\frac{U_{3nk-6k}}{U_{3k}} + (V_{2k}+q^k)q^{nk-k}U_{nk+k} \Big] + \frac{27q^{2k}}{U_k V_k (p^2-4q)^5} \Big[\frac{U_{3kn}}{U_{2k}} + (V_{2k}+q^k)V_k q^{nk-k}\frac{U_{nk-2k}}{U_k} - \frac{q^{nk-2k}U_{nk+3k}}{U_k} - \frac{q^{nk+3k}U_{nk-6k}}{U_{2k}} - (n-2)(V_{2k}+q^k)q^{nk-2k}V_{nk} \Big].$$

Hence, on page 12, line 11, line 13, line 15 and line 17 should be, respectively

$$\sum_{a+b=n} F_a^2 F_b^2 = \frac{[2nF_{n-1} + (n-1)F_n]L_n}{25} + \frac{2}{25}\left(F_{2n} + n(-1)^n\right).$$

$$\sum_{a+b=n} F_a^3 F_b^3 = \frac{2(n-1)F_{3n} + nF_{3n-3} + 18(-1)^n nF_{n-1} + 9(-1)^n (n-1)F_n}{125} + \frac{6[F_{3n} + 2(-1)^n F_n]}{125}$$

$$[2nF_{n-1} + (n-1)F_n]L_n + 2(F_{2n} + n(-1)^n) \equiv 0 \pmod 5, \quad n \geq 1,$$

$$2(n-1)F_{3n} + nF_{3n-3} + 18(-1)^n nF_{n-1} + 9(-1)^n(n-1)F_n + 6[F_{3n} + 2(-1)^n F_n] \equiv 0 \pmod{125}, \quad n \geq 1.$$

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