THE "GOLDEN RATIO" AND THE FIBONACCI NUMBERS IN THE WORLD OF ATOMS

J. WLODARSKI

In the world of atoms there are four fundamental asymmetries. They appear

- In the structure of atomic nuclei of protons and neutrons,
- In the distribution of fission fragments by mass number resulting from the bombardment of most heavy nuclei by thermal neutrons,
- In the distribution of numbers of isotopes of even stable elements,
- In the distribution of emitted particles in two opposite directions at "weak" nuclear interactions.

It turns out that the numerical values of all these asymmetries are equal approximately the "golden ratio" ("g. r.") and the numbers forming these numerical values are sometimes Fibonacci or "near"-Fibonacci numbers as follows:

1. The number of protons Z in the lightest stable nucleus as a rule is equal the number of neutrons N. When the atomic number Z increases, the proton-neutron ratio in the nucleus Z/N decreases to about 0.6.

A practical stable nucleus, found in nature may possess a maximum of 92 protons and 146 neutrons (nucleus ${}_{92}U^{238}$). The ratio of both these numbers Z/N is equal to 0.630 and differs from the "g. r."-value (if we limit the "g.r."-value to three decimals behind the point) by 0.012 only.

2. It is known that symmetrical fission of most heavy nuclei by slow neutrons is very rare. For example, in the case of ${}_{92}U^{235} + {}_{0}n^{1}$ the atomic mass of fission-fragments A = 118 happens in only about 0.01 of all cases. The most common event in this case is a splitting into two fragments with mass numbers in the range 89 – 99 and 144–134 respectively. The mass numbers 89 and 144 appearing in the nuclear reaction

$$ {}_{92}\overset{235}{U}_{143} + {}_{0}\overset{1}{n}_{1} \rightarrow {}_{36}\overset{89}{Kr}_{53} + {}_{56}\overset{144}{Ba}_{88} + 3({}_{0}n^{1}_{1}) $$

belong to two neighboring terms in the Fibonacci sequence. The ratio of 89/144 = 0.618056··· yields one of the best approximations to the "g. r."-value found in nature. The same ratio of 89/144 in the world of plants yields, for example, the distribution of sees-spirals on the disk of the sunflower.

It is interesting that the number of protons and neutrons of fission-fragments in above nuclear reaction is also one of the Fibonacci or "near"-Fibonacci numbers as the following table shows.

Table 1

Nucleon-numbers	Compound Nucleus	Fission Fragments		Terms of Fibonacci Sequence		
number of protons	92	36	56	89	34	55
number of neutrons	144	53	88	144	55	89
mass-number	236	89	144	233	89	144

Remarks:

- We have to consider the amount of 3 (on the average) emitting neutrons
- A variety of other pairs of nuclei, as Kr - Ba pair, may be produced in the above nuclear reaction, but this pair is one of the most abundant.

3. When the atomic number Z of elements in the periodic table increases, the number of isotopes of even stable elements also increases little by little and reaches a maximum (10 isotopes) at Z = 50. Behind Z = 50 the number of isotopes of even elements gradually decreases with the advancing atomic number. With that the whole row of stable elements in the periodic table is divided in the ratio 32 : 50 = 0.640. The last value differs from the "g. r."-value by 0.022 and
4. The recent (1957) discovery of parity non-conservation at "weak" interactions showed that:
   a. The β-decay of polarized neutrons is a process with an asymmetrical feature. The result of an experiment decisive for violation of the parity principle at the "weak" interactions was as follows: [1]

$$\frac{\text{Intensity of } \beta\text{-emission parallel to neutron spin}}{\text{Intensity of } \beta\text{-emission antiparallel to neutron spin}} = 0.62 \mp 0.10 \tag{1}$$

This ratio lies in the range of the "g. r."-value.

b. Various types of the hyperon-decay are also processes with an asymmetrical feature. A series of experiments was performed at some laboratories in order to investigate the distribution of emitted particles. Thereby it was found that upward emission prevailed over downward emission.

In the following table, some data of these experiments (colums 2, 3) and out of that computed values (columns 4, 5) are given:

Table 2

Experiment No.	The Emitted Particles: Number in the direction		Ratio of d/up (roughly)	Divergence of the Ratio of d/up From "g. r."-value by (roughly)
	down	up		
1 [2]	138	215	0.642	0.024
2 [3]	81	129	0.628	0.010
3 [4]	105	158	0.665	0.047

Finally it would appear that a Nobel Prize-winning English chemist and physicist F. W. Aston [5] probably was the first who showed the appearance of the Fibonacci numbers in the world of atoms. He observed that all the atoms with atomic number Z in the range 1 to 30 have the gaps representing the mass numbers of atoms which either entirely are non-existent in nature or too rare to be found. If one takes the recurring series 2, 3, 5, 8, 13, 21, 34, 55, ··· then the first 7 of these terms correspond to the missing mass numbers, but the relation breaks down at Mn^{55} and again at Y^{89}.

REFERENCES

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4. Phys. Rev. 108, 1353 (1957)
5. F. W. Aston, Isotopes (second Edition, 1924)