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For the mathematical historian interested in biographical details, Leonardo Pisano, better known to readers of this journal as Fibonacci, was a frustratingly modest genius. In his extant writings he tells us next to nothing of himself. In only one place, the second paragraph of the 1228 edition of his revised Liber Abbaci (Book of Calculation), first published in 1202, does he convey to us information about his earlier life; and even then the information, given merely as an incidental backdrop for his explanation of his purpose in writing the Liber, is very scanty and lamentably lacking in the precision which he displays in his mathematical elucidations. This second paragraph had, in the 1202 edition, been placed at the very beginning of the book; but in the revised, second edition of 1228, Leonardo wrote a dedication to the celebrated court astrologer of Frederick II, Michael Scott, who had requested a copy of the work, and thus this dedication became the work's first paragraph, with the "autobiographical paragraph" following immediately after it. Today's mathematicians are familiar with only this second, revised edition, since it is the one which Baldassare Boncompagni printed as Volume 1 of his two-volume Scritti di Leonardo Pisano (Rome, 1857-1862). Although Boncompagni knew of the existence of six manuscripts containing this autobiography, he based his edition — the first, and still the only complete printed edition which we possess — on only one manuscript, the handsome but frequently badly faded Conventi Soppressi C. I. 2616, dated to the early fourteenth century. This manuscript is now housed in the Biblioteca Nazionale Centrale in Florence; for convenience, I shall hereafter refer to itas Boncompagni's manuscript.

His failure to collate his manuscripts and his reliance upon a manuscript often difficult correctly to read led Boncompagni into an astonishing number of errors, both of transcription and of punctuation. The brief autobiographical second paragraph is unfortunately not immune from either type of error; yet this section forms the basis for most of the statements about Leonardo's early life which are found in current histories of mathematics, encyclopedias, and special articles. Unfortunately, there has also been a considerable amount of embroidering upon Leonardo's spare Latinity by many of those who have employed Boncompagni's text — which is to say all scholars who during the past eleven decades have written on Leonardo's life. It is not my intention here to refute point-by-point the many extravagant statements found about Leonardo in this more than century-old literature. Instead, I wish to present the second paragraph anew, basing my text on a collation of the six manuscripts which contain it. Following the text I shall offer a translation, along with some footnotes, keyed both to the Latin text and to the translation. Let me state at once that not all the problems in this paragraph are hereby forever resolved. I hope only that some misconceptions about

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Fibonacci can be laid aside and that we can more accurately assess what his Latinity does allow us to assert.

Cum genitor meus a patria publicus scriba¹ in duana bugee pro pisanis mercatoribus ad eam confluentibus constitutus preesset, me in pueritia mea ad se venire faciens, inspecta utilitate et commoditate futura, ibi me studio abbaci per aliquot dies² stare voluit et doceri. Vbi ex mirabili magisterio in arte³ per novem figuras indorum introductus, scientia artis in tantum mihi pre ceteris placuit, et intellexi ad illam⁴ quod quicquid studebatur ex ea⁵ apud egyptum, syriam, greciam, siliciam, et provinciam cum suis variis modis, ad que loca negotiationis causa⁷ postea⁶ peragravi per multum studium et disputationis didici conflictum⁸. Sed hoc totum etiam, et algorismum atque artem pictagore⁹ quasi errorem computavi respectu modi indorum. Quare, amplectens strictius ipsum modum indorum et attentius studens in eo, ex proprio sensu quedam addens et quedem etiam ex subtilitatibus euclidis geometrice artis apponens, summam huius libri, quam intelligibilius potui, in quindecim capitulis distinctam componere laboravi, fere omnia que inserui certa probatione ostendens, ut extra perfecto pre ceteris modo hanc scientiam¹⁰ appetentes instruantur, et gens latina¹¹ de cetero, sicut hactenus, absque illa minime inveniatur. Si quid forte minus aut plus iusto vel necessario intermisi, mihi deprecor indulgeatur, cum nemo sit qui vitio careat et in omnibus undique sit circumspectus.¹²

After my father's appointment by his homeland as state official¹ in the customs house of Bugia for the Pisan merchants who thronged to it, he took charge; and, in view of its future usefulness and convenience, had me in my boyhood come to him and there wanted me to devote myself to and be instructed in the study of calculation for some days². There, following my introduction, as a consequence of marvelous instruction in the art³, to the nine digits of the Hindus, the knowledge of the art very much appealed to me before all others, and for it⁴ I realized that all its aspects⁵ were studied in Egypt, Syria, Greece, Sicily, and Provence, with their varying methods; and at these places thereafter⁶, while on business⁷, I pursued my study in depth and learned the give-andtake of disputation⁸. But all this even, and the algorism, as well as the art of Pythagoras⁹ I considered as almost a mistake in respect to the method of the Hindus. Therefore, embracing more stringently that method of the Hindus, and taking stricter pains in its study, while adding certain things from my own understanding and inserting also certain things from the niceties of Euclid's geometric art, I have striven to compose this book in its entirety as understandably as I could, dividing it into fifteen chapters. Almost everything which I have introduced I have displayed with exact proof, in order that those further seeking this knowledge, with its pre-eminent method¹⁰, might be instructed, and further, in order that the Latin¹¹ people might not be discovered to be without it, as they have been up to now. If I have perchance omitted anything more or less proper or necessary, I beg indulgence, since there is no one who is blameless and utterly provident in all things.¹²

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1. This unsatisfactory translation is the most that should be advanced for <u>publicus</u> <u>scriba</u>, I feel. Its vagueness matches the vagueness of the Latin. We simply do not know the precise nature of the position held by Leonardo's father. He was appointed (<u>constitutus</u>) by Pisa to this post, which certainly involved duties at Bugia (present-day Bugie in Algeria) in connection with the Pisan <u>duana</u>, a word which we perhaps translate too easily as customshouse. The text as it stands offers no basis for much of the standard lore found in biographies of Leonardo regarding his father as "secretary," "merchant," "agent," "business man," "head of a factory," "warehouse head," etc.

2. Note that Leonardo says specifically that his father <u>wanted</u> him to be instructed for <u>some days</u> in the study of calculation. The phrase <u>per aliquot dies</u>, which looks like a rendering of the Italian per qualche giorno, is vague indeed, but it would be generous to consider it to imply more than a fortnight. Further, this was the period of time Leonardo's father <u>wanted</u> him to study the "abacus." How much time he <u>actually</u> spent at Bugia in his study Leonardo does not tell us. Finally, it should be noted that Leonardo uses the word <u>abbacus</u> for "calculation." By the twelfth century, in the latter part of which Leonardo was born, the older meaning of abacus as a calculation board hadgrown to include the operations which the abacus performed, namely calculation in general.

3. Just who gave Leonardo this "marvelous instruction" is not stated. It has been frequently assumed that his instructor was Moorish, but there is no hint of this in the text.

4. My translation is the best I have been able to do with <u>ad illam</u>, which I strongly suspect is corrupt, though all the manuscripts have it. As it stands, <u>illa</u> must refer to either <u>scientia</u>, the knowledge of the Hindu system, or to <u>ars</u>, the art of its exposition; but <u>ad illam</u> as a shorthand way of saying <u>ad illam cognoscendam</u> or <u>discendam</u> ("for learning it") is very harsh, and the loss of the gerundive early in the manuscript tradition is a strong probability.

5. The difficult <u>quicquid studebatur ex ea</u>, coming immediately after the strange <u>ad</u> <u>illam</u>, compels us to refer <u>ea</u> and <u>illa</u> to the same thing; the phrase can be tortured into sense by taking "whatever was studied of it" to mean "all there was of it was studied," and hence "all its aspects were studied," as the present translation renders it. It is somewhat mystifying that Leonardo mentions these particular five regions as containing <u>all</u> aspects of the Hindu lore, when we know that he also spent time in Constantinople. Did his <u>grecia</u> embrace the Byzantine capital?

6. The word for "thereafter," <u>postea</u>, gives us no indication of the amount of time which elapsed between Leonardo's boyhood experiences in Bugia and his travels around the Mediterranean. It is very probable that he returned to Pisa and went abroad again several years later, after reaching maturity. It should not be forgotten that he was still a lad (in pueritia mea, as he says) when he came to Bugia.

7. This rendering, "while on business," is based on an examination of the six autobiographical manuscripts. Boncompagni's manuscript reads ad queloca negotiationis tam postea peragraui per multum studium et disputationis didici conflictum. With this reading, tam must modify postea, and negotiationis is genitive with loca: "...to which places of business

<u>so much later</u> I wandered, through [= in the course of?] considerable study," etc. (italics mine). This is an extremely forced rendering. <u>Tam postea</u> is bad Latin for <u>tanto postea</u>; I cannot believe Leonardo wrote it, especially since all the other manuscripts give <u>causa</u> instead of <u>tam</u>. In the ligature employed by the scribes copying Leonardo's manuscripts in the twelfth to the fourteenth century, $r\hat{s}$, <u>tam</u>, and $c\hat{a}$, <u>causa</u>, are easily confused. The phrase <u>ad que loca negotiationis causa posta</u> is, I think, Leonardo's succinct way of saying "Later, while on business at these places."

8. <u>Peragravi per multum studium</u> I have rendered as "I pursued my study in depth." The phrase possibly means that in Egypt, Syria, and the other lands he has just mentioned, Leonardo utilized the opportunities which his business trips provided to investigate the Hindu number system more thoroughly. The final phrase <u>et disputationis didici conflictum</u>, also cryptic, seems a reference to the medieval practice of discussion and debate on set topics. Leonardo, it may be surmised, sought out local scholars on his business trips and mastered not only the theoretical material of the Hindu number system, but also the method of expounding it in scholarly debate.

9. The Latin here, from <u>sed</u> to <u>pictagore</u>, is a mare's nest of difficulty which has not been adequately investigated by those who have read it. Almost certainly, to judge by the variety of readings which the manuscripts exhibit at this point, there is deep -possibly incurable — textual corruption, and my translation must rely in part on emendation. There are three principal areas of difficulty.

(1) Does <u>hoc totum</u>, "all this," refer to the disputationis conflictum at the end of the preceding sentence? Or does it have as appositive <u>algorismum</u> two words later? I doubt the latter alternative. <u>Hoc totum</u>, <u>algorismum</u>, "all this, algorismus," would almost certainly be a reference to al-Khwarizmi, the great ninth-century Arab mathematician, whose very name was corrupted to "algorism" and referred to the practice of calculating with Hindu numbers. Would Leonardo say that he regarded algorism as <u>quasi errorem</u> when compared to the methods of the Hindus? (I propose a tentative answer in the next note.) Again, Leonardo has not previously discussed <u>hoc totum</u>, <u>algorismum</u>; the <u>hoc</u> should refer to something under discussion. One is practically forced back to the preceding <u>disputationis conflictum</u>, the method of argumentation itself, which Leonardo would then be contrasting with the theoretical basis of the system of Hindu numerals. This is a poor contrast at best, and I am not happy with it.

(2) The words <u>etiam et</u> are in five of the autobiographical manuscripts but are strange. If the reading is correct, <u>etiam</u> should probably be taken with <u>hoc totum</u> (= "all this, even"), and <u>et algorismum</u> should mean "and the algorismus." Once again, would Leonardo regard this algorism as "almost a mistake" when compared with the Hindu system? If the text is kept as is, I can only believe that Leonardo intends some contrast between the Hindu system as transcribed through the Arabs and the "original" system developed in its pure form by the Hindus. Had he seen some earlier work of the Hindus in his travels which made the Arab adaptation seem inferior? Kurt Vogel in his article on Fibonacci in the Dictionary of Scientific Biography (Vol. IV, pp. 603-613), speculates,

p. 605, that Fibonacci might mean the later <u>algorismus linealis</u>, reconing with lines, but this seems unlikely. When <u>algorismus</u> is mentioned by itself, without qualifying adjective, it would have for Leonardo's readers but one reference, and that is to the Hindu system of calculation.

(3) The final phrase, <u>atque artem pictagore</u>, is the last of the three things which Leonardo regards as "almost a mistake" when compared to the Hindu system. Boncompagni's text reads <u>atque arcus pictagore</u>, a phrase which has considerably exercised the ingenuity of scholars. What, they have asked, are Pythagoras' arcs? The answer, I suspect, is "a scribal concoction." My reasons for so believing and my justification for the proposed emendation are as follows.

A. The literature on Pythagoras, so far as I have ascertained, contains no allusion to any such phrase, and since Leonardo here considers <u>pictagore</u> important enough to be classified alongside the <u>algorismus</u>, discussed above, it is logical to assume that he is making a reference to some large category of Pythagorean mathematics which parallels the <u>algorismus</u>. A reference to the "arcs of Pythagoras" is too esotoric and restricted, even if Leonardo (and presumably, his readers) knew something about Pythagoras which we today do not.

B. Of the six autobiographical manuscripts, only Boncompagni's clearly reads arcus, written in ligature dre by the scribe. One other manuscript, the Biblioteca Laurenziana No. 783, written at least a century later than Boncompagni's, reads a/2, which could stand for arcus, though in extensive checking elsewhere I have found the long <u>us</u> ending for fourth declension nouns such as arcus and gradus written out by the scribe. The other four manuscripts all omit the word arcus; three have atque pictagore, one (obviously guilty of a slip) adque pictagore. It should be noted that two of these are roughly contemporary with Boncompagni's manuscript and that the latter has no special claim to paleographic superiority.

C. To balance <u>algorismum</u>, a noun is needed between <u>atque</u> ("and also") and <u>pictagore</u> ("of Pythagoras"). In the four "noun-less" manuscripts, which on other grounds appear to belong to a common tradition, it seems obvious that for some reason the word after <u>atque</u> dropped out early. Could this word have been <u>arcus</u>? In manuscript, the two words would have appeared as $\partial_{\uparrow} q_{3} \partial_{\uparrow} c^{2}$; I find it difficult to believe that some early scribe would have carelessly omitted a relatively uncommon word like <u>arcus</u>. He might, however, have been guilty of haplography if he had found $\partial_{\uparrow} q_{3} \partial_{\uparrow} q_{\hat{e}}$, <u>atque artem</u>, since both words are common (the word <u>ars</u> appears thrice in this paragraph) and in manuscript more closely resemble each other than do <u>atque arcus</u>.

D. The scribe of Boncompagni's manuscript, moreover, has already shown himself to be guilty of confusing <u>c</u> and <u>t</u> when he read $\ell \hat{a}$ as <u>tam</u> instead of <u>causa</u>. Hence it is possible that, finding something like $art\hat{e}$, he read $arc\hat{c}$, hence arcus.

E. Certainly <u>artem pictagore</u>, "the art of Pythagoras," makes excellent sense in context, balancing as it does the earlier mention of the <u>ars</u> of the Hindus and the immediately following mention of the ars of Euclid. It also serves as a satisfactory

balance to algorismum, if the interpretation of the word which I have given above is accepted.

F. I propose, then, <u>artem</u> instead of the <u>arcus</u> of Boncompagni's text, as a more reasonable, though - I freely admit - by no means certain reading. <u>Arcus</u>, however, should be given a decent burial, since both logically and paleographically it is unworthy of serious consideration.

10. The Latin here, from <u>ut</u> through <u>scientiam</u>, is rather murky, and the manuscripts admit considerable variation. However, three of the autobiographical manuscripts have Boncompagni's reading, and I have kept it, though other interpretations of the text than the one my translation implies are possible.

11. Leonardo's name for the Italians.

12. To me, this last sentence might well serve as a motto for scholars who write books. Leonardo's humility graces his genius.

[Continued from page 90.]

A PRIMER FOR THE FIBONACCI NUMBERS

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