Lack of Uniqueness — Predicting the Number of Different Summations

Can you foretell the number of different summation representations of our type, each having k terms, and leading to the same Fibonacci number F_n? Using relationship (15), our prediction becomes:

If set T is defined by

$$T = \left\{ t : 4 \le t \le \frac{n-3}{k-1} \right\},$$

then the numerosity of T, that is, the number

$$\left[\frac{n-3}{k-1}\right] - 3 \tag{16}$$

predicts the possible number of different summations of our type, each having k terms and leading to the Fibonacci number F_n.

To illustrate, there will be 52 ten-term summations of our kind leading to F_{500}. We would have:

$$\sum_{i=0}^{9} \binom{9}{i} F_{54}^{9-i} F_{55}^{i} F_{5+i} = \sum_{i=0}^{9} \binom{9}{i} F_{53}^{9-i} F_{54}^{i} F_{14+i} = \sum_{i=0}^{9} \binom{9}{i} F_{52}^{9-i} F_{53}^{i} F_{23+i}$$

$$= \cdots = \sum_{i=0}^{9} \binom{9}{i} F_{3}^{9-i} F_{4}^{i} F_{464+i} = F_{500}.$$

[Continued from page 62.]

then $V_n = L_n$, the Lucas sequence, and so (III) now gives the correct expression for (9) in (*).

Case 2. $A + B = 0$. We now obtain from (II)

$$\frac{f(x + c_1) - f(x + c_2)}{c_1 - c_2} = \sum_{n=0}^{\infty} \frac{U_n}{n!} D^n f(x), \tag{IV}$$

where $U_0 = 0$, $U_1 = 1$, and $U_{n+2} = PU_{n+1} - QU_n$. Thus for $P = 1$, $Q = -1$, $U_n = F_n$; and for $P = 2$, $Q = -1$, $U_n = P_n$, the Pell sequence. For $m = 1, 2, \cdots$, we obtain from (IV)

$$\frac{f(x + c_1^m) - f(x + c_2^m)}{c_1 - c_2} = \sum_{n=0}^{\infty} \frac{V_{mn}}{n!} D^n f(x). \tag{V}$$

Remarks. The same ideas in (*) show that the generating function of the moments of the inverse operator

[Continued on page 84.]