$$[(n - j + 1)!(n - j + 2)! \cdots (n + j - 1)!]^2 .$$

Similarly, the products of the b's in III and VI, respectively, are $[(r + j)!]^{j+1}$ and $[(r - j)!]^{j+1}$, and the product of the b's in I, II, IV and V and not in III and VI is

$$[(r - j + 1)!(r - j + 2)! \cdots (r + j - 1)!]^2 .$$

Finally, the products of the c's in II and V, respectively, are $[(n - r - j)!]^{j+1}$ and $[(n - r + j)!]^{j+1}$ and the product of the c's in I, III, IV and VI and not in II and V is

$$[(n - j - r + 1)!(n - j - r + 2)! \cdots (n + j - r - 1)!]^2 .$$

Therefore, the product of the coefficients in question is a rational square and, since the product is a product of integers, it is also an integral square as claimed.

REFERENCE

1. V. E. Hoggatt, Jr., and Walter Hansell, "The Hidden Hexagon Squares," Fibonacci Quarterly, Vol. 9 (1971), pp. 120, 133.

THE BALMER SERIES AND THE FIBONACCI NUMBERS

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In 1885, J. J. Balmer discovered that the wave lengths (λ) of four lines in the hydrogen spectrum (now known as "Balmer Series") can be expressed by the multiplication of a numerical constant $k = 364.5$ nm (1 nm = 1 nanometre = 10^{-9} m) by the simple fractions as follows:

(1) $$656 = \frac{9}{5} \times 364.5$$

(2) $$486 = \frac{4}{3} \times 364.5 = \frac{16}{12} \times 364.5$$

(3) $$434 = \frac{25}{21} \times 364.5$$

(4) $$410 = \frac{9}{8} \times 364.5 = \frac{36}{32} \times 364.5 .$$

By extending both fractions, 4/3 and 9/8, he recognized the successive numerators as the squares 3^2, 4^2, 5^2 and 6^2, and the denominators as the square-differences $3^2 - 2^2$, $4^2 - 2^2$, $5^2 - 2^2$, $6^2 - 2^2$.

From this he developed his famous formula:
[Continued on page 540.]