Since $(a,b) = 1$ the second equation of (12) yields either

(14) $$b = \pm t^2, \qquad a = \mp 5a^2$$

or

(15) $$b = \pm 5t^2, \qquad a = \mp s^2\ .$$

Equations (13) and (14) yield

$$(\mp 10s^2 \pm t^2)^2 - 5t^4 = 4.$$

By (5), the only integer solutions of this equation occur for $t = 0$, 1 or 12. But none of these values of t yield a value for s. Equations (13) and (15) yield

$$(\mp 2s^2 \pm 5t^2)^2 - 125t^4 = 4.$$

By Lemma 2, $t = 0,\ s = 1,\ a = \pm 1,\ b = 0,\ L_n = 1.$ The proof is complete.

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[Continued from P. 339.]

Since

$$(a/-1) = (b/-1) = 1,$$

therefore

$$\begin{aligned}(-a/-b)(-b/-a) &= (a/b)(b/a)(-1/a)(-1/b)\\ &= ((-1/a)/(-1/b))(-1/a)(-1/b)\\ &= 1\end{aligned}$$

if and only if

$$(-1/a) = (-1/b) = 1\ .$$

Therefore,

(4) $$(-a/-b)(-b/-a) = -((-1/-a)/(-1/-b)).$$

From (1), (2), (3) and (4), it can be seen that the theorem is true for all sixteen combinations of

$$(a/-1) = \pm 1, \qquad (b/-1) = \pm 1, \qquad (-1/a) = \pm 1 \quad \text{and} \quad (-1/b) = \pm 1.$$

Corollary 1. If $a \equiv 0$ or 1 (mod 2), $b \equiv 1$ (mod 2) and $(a,b) = 1$, and if $a_1 \equiv a_2$ (mod b), then

$$(a_1a_2/b) = \left(\frac{(a_1a_2/-1)}{(b/-1)}\right)\ .$$

In other words, $(a_1a_2/b) = 1$ if and only if a_1a_2 is positive and/or b is positive.

[Continued on P. 344.]