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ON THE HARRIS MODIFICATION OF THE EUCLIDEAN ALGORITHM

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V. C. Harris[1] (see D. E. Knuth[2] also) modified the Euclidean algorithm (= algorithm by greatest integers) for finding the gcd of two odd integers $a > b > 1$. The conditions $a = bq + r$, $|r| < b$, $2|r$ define the integers q,r uniquely. In case $r = 0$, stop. In case $r \neq 0$, divide r by its highest power of 2 and obtain c (say); proceed with b, $|c|$ instead of a,b. Denote by $H(a,b)$ the number of steps in this Harris algorithm.

Example: $83 = 47 \cdot 1 + 4 \cdot 9$, $47 = 9 \cdot 5 + 2 \cdot 1$, $9 = 1 \cdot 9$; $H(83,47) = 3$.

Denote by $E(a,b)$ resp. $N(a,b)$ the number of steps in the algorithm by greatest resp. nearest integers for $a > b > 0$. According to Kronecker, $N(a,b) \leqslant E(a,b)$ always. In this note we prove that $H(a,b)$ is sometimes much larger than $E(a,b)$ and sometimes much smaller than $N(a,b)$.

Let

$$c_0 := 1, \quad c_{n+1} = 2c_n + 5 \quad (n \geqslant 0);$$

obviously

$$E(c_{n+1}, c_n) \leqslant 5 \quad (n \geqslant 0).$$

Furthermore, since

$$c_{n+2} = 3c_{n+1} - 2c_n, \quad 2 \nmid c_n \quad (n \geqslant 0),$$

the choice $a_k = c_k$, $b_k = c_{k-1}$ $(k > 0)$ gives

Theorem 1. For every integer $k > 0$ there exist odd integers $a_k > b_k > 0$ with

$$E(a_k, b_k) \leqslant 5, \quad H(a_k, b_k) = k.$$

Let

$$v_0 := 0, \quad v_1 := 1, \quad v_n := 2v_{n-1} + v_{n-2} \quad (n > 1);$$

then

$$(v_{n+1}, v_n) = 1, \quad v_n \leqslant 3^{n-1}, \quad 2|v_n \Leftrightarrow 2|n \quad (n \geqslant 0).$$

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[1] *The Fibonacci Quarterly,* Vol. 8, No. 1 (February, 1970), pp. 102–103.

[2] *The Art of Computer Programming,* Vol. 2, "Seminumerical Algorithms," Addison-Wesley Pub., 1969, pp. 300, 316

[continued on page 200.]