we get

$$\delta_a^n = \sum_{m=0}^{\infty} \frac{n!}{m!}T(m,\ n)a^m D^m, \qquad a^n D^n = \sum_{m=0}^{\infty} \frac{n!}{m!}t(m,\ n)\delta_a^m,$$

$$\delta_a^n = \sum_{m=0}^{\infty} \frac{n!}{m!}K(m,\ n,\ r)\delta_b^m,\ r = a/b.$$

Finally, let

$$Q_m(z;\ s) = \sum_{x=0}^{z} (sx)^{[m]}$$

and put

$$Q_{2m}(z;\ s) = \frac{2z+1}{2}\sum_{n=0}^{m} \frac{Q_{m,n,s}}{2n+1}\ \frac{(z+n)!}{(z-n)!}.$$

Then

$$(sx)^{[2m]} = \sum_{n=0}^{m} Q_{m,n,s}\frac{x(x+n-1)!}{(x-n)!} = \sum_{n=0}^{m} Q_{m,n,s}x^{[2m]},$$

and by (2.10),

$$Q_{m,n,s} = K(2m,\ 2n,\ s).$$

A similar expression may be obtained for $Q_{2m+1}(z;\ s)$.

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ON THE FIBONACCI NUMBERS MINUS ONE

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Let A be the $n \times n$ matrix with elements defined by

$$a_{ij} = -1 \text{ if } i = j - 1;\ 1 + \mu \text{ if } i = j;\ -\mu \text{ if } i = j + 2;$$

and 0 otherwise. If $n \geq 3$ and μ is a positive number, then A is a special case of a matrix that was shown in [1] to be useful in the design of two-up, one-down ideal cascades for uranium enrichment. The purpose of this paper is to derive certain properties of the determinant D_n of A and to point out its relation to the Fibonacci numbers.

Expansion of the determinant of A according to its first column leads to the recurrence relation

(1) $\quad D_1 = 1 + \mu,\ D_2 = (1 - \mu)^2$, and $D_n = (1 + \mu)D_{n-1} - \mu D_{n-3}$ for $n \geq 3$.

For convenience, set $D_0 = 1$.

By using standard techniques for generating functions, it can be shown that the generating function $D(x)$ for $\{D_n\}$ (with positive radius of convergence) is

(2) $$D(x) = [1 - (1 + \mu)x + \mu x^3]^{-1} = \sum_{i=0}^{\infty} \sum_{j=0}^{i} (-1)^j \binom{i}{j} \mu^j (1 + \mu)^{i-j} x^{i+2j}.$$

Therefore, an explicit expression for D_n is

(3) $$D_n = \sum_{k=0}^{[n/3]} (-1)^k \binom{n-2k}{k} \mu^k (1+\mu)^{n-3k},$$

where $[n/3]$ denotes the integral part of $n/3$.

Adding the recurrence relations (1) for $n = 3, 4, 5, \ldots, m$ leads, on simplification, to the alternative recurrence relation

(4) $$D_m - D_{m-1} - D_{m-2} = 1 \text{ for } m \geq 3.$$

The homogeneous equation corresponding to (4) has the linearly independent solutions

$$g_m(\mu) = \left(\frac{\mu + \sqrt{\mu^2 + 4\mu}}{2}\right)^m, \quad h_m(\mu) = \left(\frac{\mu - \sqrt{\mu^2 + 4\mu}}{2}\right)^m, \text{ for all } m \geq 3,$$

and a particular solution of (4) is

$$p_m(\mu) = \begin{cases} 1/(1 - 2\mu) & \text{if } \mu \neq 1/2, \\ 2m/3 & \text{if } \mu = 1/2. \end{cases}$$

Hence, the general solution of (4) is of the form

(5) $$D_m = c_1 g_m + c_2 h_m + p_m \text{ for } m \geq 3,$$

where c_1 and c_2 are constants that can be determined from (1).

In the special case when $\mu = 1$, let Δ_m denote the determinant of the matrix A. Then (3), (4), and (5), respectively, become

$$\Delta_n = \sum_{k=0}^{[n/3]} (-1)^k \binom{n-2k}{k} 2^{n-3k}, \quad n \geq 0,$$

$$\Delta_m - \Delta_{m-1} - \Delta_{m-2} = 1, \quad m \geq 3,$$

and

$$\Delta_m = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{m+3} - \left(\frac{1-\sqrt{5}}{2}\right)^{m+3}\right] - 1, \quad m \geq 3.$$

It is clear that the members of the sequence $\{\Delta_m\}$ are the Fibonacci numbers minus one [2] and that the results for $\mu \neq 1$ generalize those for $\mu = 1$.

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EDITOR'S NOTE:

Selecting the names of those individuals who were asked to submit manuscripts for the Memorial Issue was not an easy task on the part of the Board of Directors and Herta Hoggatt. Vern knew and worked with so many of you that it would have been impossible to ask all of you. As the editor, I apologize for any oversights. Furthermore, Mrs. Herta Hoggatt and family wish to express their sincere appreciation to all of those authors who contributed to the Memorial Issue.

—Gerald E. Bergum