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WHY ARE 8:18 AND 10:09 SUCH PLEASANT TIMES?

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To rephrase this facetious question: Why does a watch or a clock appear most pleasing when its hands are set at approximately 8:18 or 10:09? In case the reader has not noticed, nondigital watches and clocks (not running) on display in stores, or photographs of them in catalogs, often are set very nearly at one of these two times. One common myth concerning the time 8:18 (or 8:17) is that this is precisely the time at which President Abraham Lincoln died. In [1, p. 394], this myth is discussed. In reference to clock faces painted on signs, it is suggested that 8:17 is used for the setting of the hands to allow more space on the clock face for advertising.

The purpose of this note is to investigate the aforementioned question. In the process, interesting relationships between these two times and the golden ratio will be discovered.

First, one observes that at both 8:18 and 10:09 the angle between 12 o'clock and the hour hand is approximately equal to the angle between 12 o'clock and the minute hand. Of course, 8:20 and 10:10 would be "equal-angled" if the hour hnad moved in discrete hourly jumps rather than moving continuously. Certainly, then, symmetry plays a key role.

Theorem

For the times listed in the table, the clock hands are approximately "equal-angled."

Proof: The conclusion can be drawn by observing a clock or by using the following analysis. Let α be the angle formed by 12 o'clock and the minute hand and β the angle formed by 12 o'clock and the hour hand. Then, since each hour yields 30°,

$$\beta = (\alpha/360°)30° = \alpha/12. \qquad (*)$$

For equal angles,

$360° - \beta$ differs from α by some integral multiple of $360°$;

hence

$$\alpha + \beta = 360°k. \qquad (**)$$

From (*) and (**), $\alpha = 12 \cdot 360°k/13$.

Now, the hour, h, is $[\beta/30°]$ (30° per hr.), the minute, m, is $[5\alpha/30°] - 60h$ (30° per 5 min.), and the second is 60 times the "decimal part" of m. The following table, generated by varying k, lists hours, minutes, seconds and, most importantly, the angle β. Note that all of the angles have been reduced to $<90°$ so that, for half of the listed times, the angle is measured with respect to 6 o'clock, e.g., 5:32.

TABLE

Hour	Minute	Second	Angle (in degrees)
12	55	23	27.7
1	50	46	55.4
2	46	9	83.1
3	41	32	69.2*
4	36	55	41.5*
5	32	18	13.8*
6	27	42	13.8*
7	23	5	41.5*
8	18	28	69.2*
9	13	51	83.1
10	9	14	55.4
11	4	37	27.7

*Measured with respect to 6 o'clock.

The time 8:18 will be investigated first; for this, the following lemma will be useful.

Lemma

Let φ be the golden ratio, $a = \text{Arctan}\,\varphi$, and $b = 90° - a$; then,

$$\tan 2b = 2.$$

Proof: Since $\tan a = \varphi$, $\tan^2 a - \tan a - 1 = 0$; hence, division by $\tan a - \cot a = 1$. Then,

$$\tan 2b = \frac{2 \tan b}{1 - \tan^2 b} = \frac{2}{\cot b - \tan b} = \frac{2}{\tan a - \cot a} = \frac{2}{1} = 2.$$

Note: $a \approx 58.3°$.

Now, if one were to visualize a rectangle (see Figure 1) superimposed on a clock face at the time 8:18 (or at 3:41 when the hands are reversed) using the hour and the minute hands to form semidiagonals, one would see a rectangle whose corners were approximately at minutes 12, 18, 42, and 48. At these particular times, β ($\approx 69.2°$) is very nearly $2b$ ($\approx 63.4°$); in fact, the relative error,

$$\frac{69.2° - 63.4°}{63.4°}$$

is less than 10%. From the lemma, it follows that the imagined rectangle is approximately proportioned 2 to 1. That is, the rectangle would (almost) be formed by two squares. By checking the table, one can see that 8:18 (and 3:41) give the "equal-angle" times for which the imagined rectanble most closely approximates such a rectangle.

The imagined rectangle at 10:09 (or 1:50) is even more significant. If one were to visualize a rectangle (see Figure 2) at these times, one would see a rectangle whose corners were approximately at the minutes 9, 21, 39, and 51. At these particular times, β ($\approx 55.4°$) is very nearly Arctan φ ($\approx 58.3°$); in fact, the relative error,

$$\frac{58.3° - 55.4°}{58.3°}$$

is less than 5%. Therefore, at 10:09 (and 1:50), the imagined rectangle is approximately a golden rectangle. A check of the table shows that these times give the "equal-angle" times for which the imagined rectangle most closely approximates the golden rectangle.

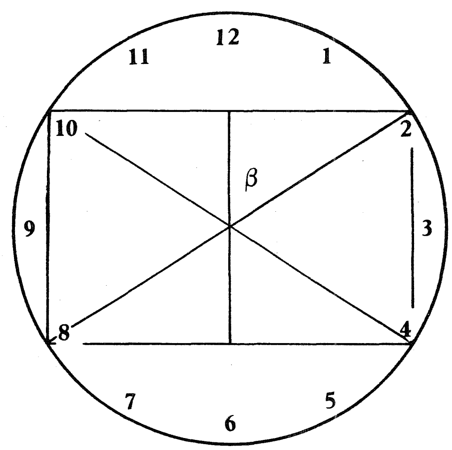

FIGURE 1

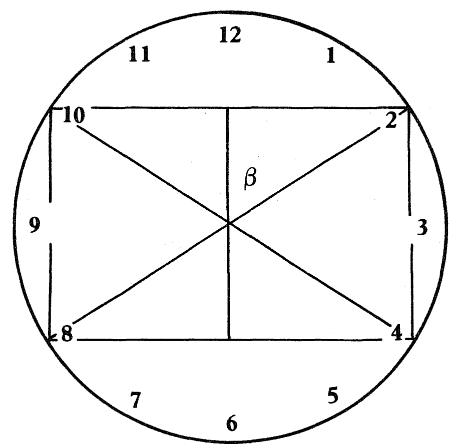

FIGURE 2

Perhaps the close association with the golden ratio for 8:18 and the good approximation to the golden rectangle for 10:09 are the reasons why these two times are chosen for display purposes.

REFERENCE

1. Lloyd Lewis. *Myths after Lincoln.* New York: Harcourt, Brace, 1929.

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