Juan B. Gil, Michael D. Weiner and Catalin Zara
Complete Padovan sequences in finite fields,
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Abstract

Given a prime $p \geq 5$, and given $1 < \kappa < p-1$, we call a sequence $(a_n)_n$ in $\mathbb{F}_p$ a Φ_κ-sequence if it is periodic with period $p-1$, and if it satisfies the linear recurrence $a_n + a_{n+1} = a_{n+\kappa}$ with $a_0 = 1$. Such a sequence is said to be a complete Φ_κ-sequence if in addition $\{a_0, a_1, \ldots, a_{p-2}\} = \{1, \ldots, p-1\}$. For instance, every primitive root b mod p generates a complete Φ_κ-sequence $a_n = b^n$ for some (unique) κ. A natural question is whether every complete Φ_κ-sequence is necessarily defined by a primitive root. For $\kappa = 2$ the answer is known to be positive. In this paper we reexamine that case and investigate the case $\kappa = 3$ together with the associated cases $\kappa = p-2$ and $\kappa = p-3$.