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An Identity Motivated by an Amazing Identity of Ramanujan,
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Abstract

Ramanujan stated an identity to the effect that if three sequences $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ are defined by $r_1(x) =: \sum_{n=0}^{\infty} a_n x^n$, $r_2(x) =: \sum_{n=0}^{\infty} b_n x^n$ and $r_3(x) =: \sum_{n=0}^{\infty} c_n x^n$ (here each $r_i(x)$ is a certain rational function in x), then

$$a_n^3 + b_n^3 - c_n^3 = (-1)^n, \quad \text{for all } n \geq 0.$$

Motivated by this amazing identity, we state and prove a more general identity involving eleven sequences, the new identity being “more general” in the sense that equality holds not just for the power 3 (as in Ramanujan’s identity), but for each power j , $1 \leq j \leq 5$.