Jason I. Brown, Karl Dilcher, and Dante V. Manna Series Representations of Theta Functions in Terms of a Sequence of Polynomials,
Fibonacci Quart. 50 (2012), no. 1, 5–10.

Abstract

We derive series expansions for the Jacobi theta functions $\theta_j(q)$, j = 2, 3, 4, and for $\theta_3(z, q)$, all in terms of a certain sequence of sparse binomial-type polynomials. As consequences we obtain series identities involving second-order recurrence sequences and Chebyshev polynomials of the first kind.