Stephen R. Wassell and Samantha Benito
Edge-Length Ratios Between Dual Platonic Solids: A Surprisingly New
Result Involving the Golden Ratio,
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Abstract

A thorough review of the appropriate literature reveals the possibility of a fundamental ratio going unnoticed until now, or at least being absent from the literature: the edge-length ratio between a regular dodecahedron and its circumscribing dual (polar reciprocal) icosahedron when paired vertex to face, namely one third of the golden ratio. This ratio completes an elegant triplet of ratios for vertex-to-face dual pairings when the outer Platonic solid is the tetrahedron, octahedron, and icosahedron (i.e., those with triangular faces), specifically $1:3, \sqrt{2}:3$, and $\phi:3$, respectively.