Lawrence Somer and Michal Křížek Power Digraphs Modulo n are Symmetric of Order M If and Only If M is Square Free, Fibonacci Quart. **50** (2012), no. 3, 196–206.

Abstract

We assign to each pair of positive integers $k \ge 2$ and n a digraph G(n,k) whose set of vertices is $H = \{0, 1, \ldots, n-1\}$ and for which there is a directed edge from $a \in H$ to $b \in H$ if $a^k \equiv b \pmod{n}$. The digraph G(n,k) is symmetric of order M if its set of components can be partitioned into disjoint subsets, each containing exactly M isomorphic components. Deng and Yuan completely characterized all symmetric digraphs of order M when M = 2 or M is divisible by an odd prime. We demonstrate that their classification is complete by showing that there are no symmetric digraphs G(n, k) of order 2^s for $s \ge 2$.