M. Lawrence Glasser and Yajun Zhou

An Integral Representation for the Fibonacci Numbers and Their Generalization,

Fibonacci Quart. 53 (2015), no. 4, 313–318.

Abstract

We report on an integral representation for the Fibonacci sequence

$$F_n = \frac{1}{\sqrt{5}} \left(\frac{\sqrt{5}+1}{2}\right)^n - \frac{2}{\pi} \int_0^\infty \frac{\sin(x/2)}{x} \frac{\cos(nx) - 2\sin(nx)\sin x}{5\sin^2 x + \cos^2 x} \, dx$$

and give two different proofs, with or without invoking complex analysis. These proofs allow us to present some generalizations of this integral representation along two different directions.