R. S. Melham
On Certain Families of Finite Reciprocal Sums that Involve Generalized Fibonacci Numbers,
Fibonacci Quart. 53 (2015), no. 4, 323–334.

Abstract

In this paper, we find closed forms, in terms of rational numbers, for certain finite sums. Our most general results are for finite sums where the denominator of the summand is a product of terms from a sequence that generalizes both the Fibonacci and Lucas numbers.