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On the Positive Integer Points of Certain Two Parameter Families of Hyperbolas,
Fibonacci Quart. 54 (2016), no. 3, 247–252.

Abstract

In this paper, in Sections 2–4, we consider the problem of finding all the positive integer points of a certain two parameter family of quadratic Diophantine equations. Geometrically, this family of quadratic Diophantine equations is a two parameter family of hyperbolas. We proceed by linking the positive integer points of this family of hyperbolas to the positive integer points of a two parameter family of Pell equations. Along the way, we prove a theorem that gives the fundamental solution of the two parameter family of Pell equations in question. In Section 5, we summarize our findings regarding the positive integer points of another two parameter family of quadratic Diophantine equations.