R. S. Melham

Further Closed Forms for Finite Sums of Weighted Products of Generalized Fibonacci Numbers,

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Abstract

The finite sum

$$\sum_{i=1}^{n} 2^{-i} F_{i-1} = 1 - \frac{F_{n+2}}{2^n},$$

occurs in Section 9.1 of Knott [1], and is the inspiration for the present paper. We refer to the term 2^{-i} in the summand as the weight term. Here, we present seven families of such finite sums that we believe to be new. In each of these seven families, the product that defines the summand can be made arbitrarily long. The sequences that we employ are generalizations of the Fibonacci/Lucas sequences.