Arthur T. Benjamin, Joshua Crouch, and James A. Sellers
Unified Tiling Proofs of a Family of Fibonacci Identities,
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Abstract

In a recent work, Baxter and Pudwell mentioned the following identity for the Fibonacci numbers F_n and noted that it can be proven via induction: For all $n \geq 1$,

$$F_{2n} = 1 \cdot F_{2n-2} + 2 \cdot F_{2n-4} + \cdots + (n-1) \cdot F_2 + n.$$

We give a combinatorial proof of this identity and a companion identity. This leads to an infinite family of identities, which are also given combinatorial proofs.