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Primefree Shifted Binary Linear Recurrence Sequences,
Fibonacci Quart. **57** (2019), no. 1, 51–67.

Abstract

We say a sequence $\mathcal{X} = (x_n)_{n\geq 0}$ is *primefree* if $|x_n|$ is not prime for all $n \geq 0$ and, to rule out trivial situations, we require that no single prime divides all terms of $\mathcal{X}$. For $a, b, w_0, w_1 \in \mathbb{Z}$, we let $\mathcal{W}(w_0, w_1, a, b) = (w_n)_{n\geq 0}$ denote the general linear binary recurrence that is defined by

$$w_n = aw_{n-1} + bw_{n-2} \quad \text{for } n \geq 2.$$

It has been shown recently for any sequence $\mathcal{X} \in \{\mathcal{W}(0, 1, a, 1), \mathcal{W}(2, a, a, 1)\}$, that there exist infinitely positive integers k such that both of the shifted sequences $\mathcal{X} \pm k$ are simultaneously primefree, and moreover, each term has at least two distinct prime divisors. In this article, we extend these theorems by establishing analogous results for all but finitely many sequences

$$\mathcal{X} \in \{\ \mathcal{W}(0, 1, a, -1),\ \mathcal{W}(2, a, a, -1),\ \mathcal{W}(1, 1, a, -1),\ \mathcal{W}(-1, 1, a, -1)\ \},$$

which provides additional evidence to support a conjecture of Ismailescu and Shim.