Ben Delo and Filip Saidak
Euclid's Theorem Redux,
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Abstract

We discuss properties of certain recursive sequences, constructed in ways that guarantee pairwise coprimality of their iterative factors. As a direct application, we obtain a couple of new proofs of Euclid's Theorem concerning the infinitude of the prime numbers. In particular, one class of proofs of the theorem can be obtained from the following estimate:

$$\omega\left(a^n - 1\right) \geq \Omega(n),$$

which we establish for all $a \geq 2$ and $n \in \mathbb{N}$ with $\gcd(a-1, n) = 1$.