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An Introduction to Completeness of Positive Linear Recurrence Sequences,

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Abstract

A positive linear recurrence sequence (PLRS) is a sequence defined by a homogeneous linear recurrence relation with positive coefficients and a particular set of initial conditions. A sequence of positive integers is *complete* if every positive integer is a sum of distinct terms of the sequence. One consequence of Zeckendorf's theorem is that the sequence of Fibonacci numbers is complete. Previous work has established a generalized Zeckendorf's theorem for all PLRS's. We consider PLRS's and want to classify them as complete or not. We study how completeness is affected by modifying the recurrence coefficients of a PLRS. Then, we determine in many cases which sequences generated by coefficients of the form $[1, \ldots, 1, 0, \ldots, 0, N]$ are complete. Further, we conjecture bounds for other maximal last coefficients in complete sequences in other families of PLRS's. Our primary method is applying Brown's criterion, which says that an increasing sequence $\{H_n\}_{n=1}^{\infty}$ is complete if and only if $H_1 = 1$ and $H_{n+1} \leq 1 + \sum_{i=1}^{n} H_i$. This paper is an introduction to the topic that is explored further in [BHLLMT].