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Closing an Open Problem on Negative Base Happy Numbers,
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Abstract

For $b \leq -2$, let $S_{2,b} : \mathbb{Z} \to \mathbb{Z}_{\geq 0}$ be the function taking an integer to the sum of the squares of the digits of its base b expansion. An integer a is a b-happy number if there exists $k \in \mathbb{Z}^+$ such that $S_{2,b}^k(a) = 1$. It has been shown that for $b \leq -5$ and odd, there exist arbitrarily long finite arithmetic sequences with constant difference 2 of b-happy numbers and that for $b \in \{-4, -6, -8, -10\}$, there exist arbitrarily long finite sequences of consecutive b-happy numbers. In this work, we complete this result, proving that, as conjectured, for all even $b \leq -4$, there exist arbitrarily long finite sequences of consecutive b-happy numbers.