Yuji Tsuno

Polynomial Values with Integer Coefficients for the Generating Functions of Fibonacci Polynomials,

Fibonacci Quart. **62** (2024), no. 2, 157–172.

Abstract

Fibonacci polynomials are generalizations of Fibonacci numbers, so it is natural to consider polynomial versions of the various results for Fibonacci numbers. According to Hong, Pongsriiam, and Bulawa and Lee, the generating function of the Fibonacci sequence in the domain of rational numbers, $f(t) = t/(1-t-t^2)$, takes an integer value if and only if $t = F_k/F_{k+1}$ for some $k \in \mathbb{N}$ or $t = -F_{k+1}/F_k$ for some $k \in \mathbb{N}^+$, where F_k is the kth Fibonacci number. This study is built on their work by considering polynomial sequences that satisfy the recurrence relation $F_{i+2}(x) = axF_{i+1}(x) + bF_i(x)$ with initial values $(F_0(x), F_1(x)) = (0, 1)$, where a and b are positive integers such that $b|a$. As an application, for a square-free natural number $d \in \mathbb{N}$, we verify the results are of the same form as the above for the generating function of the sequence satisfying the recurrence relation $F_{i+2}(\sqrt{d}) = a\sqrt{d}F_{i+1}(\sqrt{d}) + bF_i(\sqrt{d})$ with initial values $(F_0(\sqrt{d}), F_1(\sqrt{d})) = (0, 1)$.