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Knights Are 24/13 Times Faster Than the King,
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Abstract

On an infinite chess board, how much faster can the knight reach a square compared with the king, on average? More generally, for coprime $b > a \in \mathbb{Z}_{\geq 1}$ such that $a + b$ is odd, define the (a, b)-knight and the king as

$$\mathrm{N}_{a,b} = \{(a, b), (b, a), (-a, b), (-b, a), (-b, -a), (-a, -b), (a, -b), (b, -a)\},$$

$$\mathrm{K} = \{(1, 0), (1, 1), (0, 1), (-1, 1), (-1, 0), (-1, -1), (0, -1), (1, -1)\} \subseteq \mathbb{Z}^2,$$

respectively. One way to formulate this question is by asking for the average ratio, for $\mathbf{p} \in \mathbb{Z}^2$ in a box, between $\min\{h \in \mathbb{Z}_{\geq 1} \mid \mathbf{p} \in h\mathrm{N}\}$ and $\min\{h \in \mathbb{Z}_{\geq 1} \mid \mathbf{p} \in h\mathrm{K}\}$, where $hA = \{\mathbf{a}_1 + \cdots + \mathbf{a}_h \mid \mathbf{a}_1, \ldots, \mathbf{a}_h \in A\}$ is the h-fold sumset of A. We show that this ratio equals $2(a+b)b^2/(a^2 + 3b^2)$.