## The Magic of Math: Solving for $x$ and Figuring out Why

 by Arthur BenjaminBasic Books, New York, 2015, 321 pp.
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Bottom line: This book is well written, enjoyable to read, accessible to anyone who might like mathematics, and interesting and worth reading to those of us who have a life-long mathematics love affair. The book is full of small, digestible pieces of genuine mathematics which come sometimes with full rigor, sometimes with excellent suggestions of rigor, such as "proofs without words," and sometimes just statements of fact (for which rigor would be out of place and indigestible). It is to a great extent a hard-copy version of the author's video lectures The Joy of Mathematics for The Great Courses company.

For those of us mathematics lovers (Quarterly readers), the book is a huge helping of comfort food, but some familiar dishes are served up in novel ways and, some unfamiliar ones are nice additions. We can also enjoy ourselves enumerating the book's omissions, imagining what topics we would have included. The book is so engaging that we would want to participate in its creation. Perhaps it could become a wiki-it would grow to monstrous proportions. The author's selections and decisions to stop where he did were admirable: the book is just right.

The "Magic of Infinity" was particularly interesting to me, although it had only two pages devoted to infinity (countability and Cantor's diagonal argument for the uncountability of the reals) with the bulk of the chapter devoted to infinite series and a little bit on magic squares. This chapter covers several divergent series (e.g., $1+2+3+\cdots=-1 / 12$ and $1+2+4+\cdots+2^{n}+\cdots=-1$ ), a topic I have never seen in such books. (One small quibble: proofs using " $\ldots$ " are not an alternative to mathematical induction; the ellipses are defined inductively, and their use saves a lot of repetition.)

As a part of my reviewing process, I put the book into the hands of some (otherwise good) people for whom mathematics is not in their top ten loves. At the end of their mandated 20-minute reading, they all reported that I had given them a pleasant task, that the writing style was clear and easy to read, and that they had learned new things.

An elementary school student started from page 1 and ended in "The Magic of Algebra" chapter. He explained to me how Gauss quickly determined $1+2+3+\cdots+100=5050$ and he performed the algebra magic trick using the steps of $(2 x+10) / 2-x$, divining that my answer was 5 (my secret number was $x$ ). The sequence trick first then explain with algebra was the nicest path I have seen for showing beginners the Joy of Algebra. (However, the book's subtitle, "Solving for $x$ and Figuring Out Why," misleadingly suggests the book is all about Algebra - it is not.)

A college student (humanities major) chose to read "The Magic of Nine." Now she knows about digital roots and the magic of casting out nines. "This is so neat!"

A high school student who may go on to study engineering and already finds mathematics pretty appealing found at least half of what I had him read novel and interesting. (He is not among the innumeracy: he's the one who at age seven explained the Prime Number Sieve to his mom six months after I showed it to him.)

The twelve chapters of the book are largely along the lines of the 24 half-hour lectures in the Joy of mathematics. They are: The Magic of Numbers, Algebra, 9, Counting, Fibonacci Numbers, Proofs, Geometry, $\pi$, Trigonometry, $i$ and $e$, Calculus, and Infinity. (And the book has a good index.)

The book does offer up several magic tricks, most of them based on algebra, but I think largely the term magic is used here as a synonym for joy, as in the title of the video lectures. Other applicable synonyms: beauty, surprise, wonder, astonishment, ....

The author is a humorous guy in his stage mathemagic, his lectures, and in his writing. Some of this takes the form of bad puns ("a proof by Cantor-example"), but the humor flows nicely within the narrative and never gets in the way. A pleasant personality always shows through.

The book is set up so readers can dip into it at random and be rewarded on almost every page. Within chapters the flow does build up but generally the chapters do not depend on one another. Math tyros should probably pick a random chapter and start at the beginning; Quarterly readers can pick pages arbitrarily and savor the novel ways of presenting things, the greetings of old friends, or the introduction to new friends.

Conclusion: This is a wonderful book for lots of us. It is fun for those of who know most of this material to browse, and it would be great to put in the hands of kids who would, a couple of generations ago, been inspired by Martin Gardner. I wish I had had such a book. And it would be the right book for all of us to have on our shelves to press onto our friends mystified by our love for mathematics.

