THE POWERFULL 1979

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(*A*) $1979 = 990^2 - 989^2$

(*B*) $1979 = 3^2 + 11^2 + 43^2 = 3^2 + 17^2 + 41^2$

$= 2^2 + 5^2 + 7^2 + 11^2 + 13^2 + 17^2 + 19^2 + 31^2$

(*C*) $1979 = 5^2 + 27^2 + 35^2$

$= 7^2 + 29^2 + 33^2$

$= 1^2 + 4^2 + 21^2 + 39^2$

$= 3^2 + 5^2 + 24^2 + 37^2$

$= 3^2 + 7^2 + 25^2 + 36^2$

$= 1^2 + 3^2 + 6^2 + 13^2 + 42^2$

$= 1^2 + 4^2 + 5^2 + 16^2 + 41^2$

$= 2^2 + 7^2 + 17^2 + 26^2 + 31^2$

$= 1^2 + 2^2 + 3^2 + 5^2 + 28^2 + 34^2$

$= 1^2 + 3^2 + 4^2 + 5^2 + 22^2 + 38^2$

$= 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 18^2 + 40^2$

$= 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 30^2 + 32^2$

$= 1^2 + 2^2 + 6^2 + 8^2 + 10^2 + 19^2 + 20^2 + 22^2 + 23^2$

$= 3^2 + 4^2 + 6^2 + 7^2 + 8^2 + 9^2 + 11^2 + 12^2 + 13^2 + 14^2 + 15^2 + 16^2 + 17^2 + 18^2$

These expressions, that involve the squares of all positive integers < 44, are just a few examples chosen from the multitude of partitions of 1979 into squares.

(*D*) $1979 = 2^3 + 3^3 + 6^3 + 12^3$

$= 1^1 + 13^2 + 8^3 + 6^4 + 1^5$

$= 2^0 + 2^1 + 2^3 + 2^4 + 2^5 + 2^7 + 2^8 + 2^9 + 2^{10}$

$= 2^{11} - 2^6 - 2^2 - 2^0$

$= -3^0 + 3^2 + 3^3 - 3^5 + 3^7$

$= 1^3 + 9^3 + 7^3 + 9^3 + 1^1 + 9^2 + 7^1 + 9^2 + 1 \cdot 9 \cdot 7/9$

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AN OBSERVATION CONCERNING WHITFORD'S "BINET'S FORMULA GENERALIZED"

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In [1], Whitford generalizes the Fibonacci sequence by modifying the defining equations of the Fibonacci sequence by letting

$$G_n = \frac{[(1 + \sqrt{p})/2]^n - [(1 - \sqrt{p})/2]^n}{\sqrt{p}} \qquad (n \geq 1).$$

This leads to a sequence whose defining equations are $G_1 = G_2 = 1$,

$$G_{n+2} = G_{n+1} + [(p - 1)/4]G_n \qquad (n \geq 1).$$

One can also use Whitford's Generalization of Binet's formula to obtain a generalization of the Lucas sequence. From [2], $L_n = \alpha^n + \beta^n$ $(n \geq 1)$, where $\alpha = (1 + \sqrt{5})/2$ and $\beta = (1 - \sqrt{5})/2$. By using Whitford's α and β, the Lucas sequence can be generalized by a sequence H_n, where

$$H_n = [(1 + \sqrt{p})/2]^n + [(1 - \sqrt{p})/2]^n \qquad (n \geq 1).$$

Now, since α and β satisfy $x^2 - x - [(p - 1)/4] = 0$,

$$\begin{aligned} H_{n+2} &= \alpha^{n+2} + \beta^{n+2} = \alpha^n(\alpha^2) + \beta^n(\beta^2) = \alpha^n(\alpha + [(p - 1]/4]) + \beta^n(\beta + [(p - 1)/4]) \\ &= \alpha^{n+1} + \beta^{n+1} + [(p - 1)/4](\alpha^n + \beta^n) = H_{n+1} + [(p - 1)/4]H_n. \end{aligned}$$

Furthermore, $H_1 = (1 + \sqrt{p})/2 + (1 - \sqrt{p})/2 = 1$ and

$$H_2 = [(1 + \sqrt{p})/2]^2 + [(1 - \sqrt{p})/2]^2 = (p + 1)/2.$$

Thus, the analog of Whitford's generalization of the Fibonacci sequence is the generalization of the Lucas sequence,

$$H_1 = 1,\ H_2 = (p + 1)/2,\ H_{n+2} = H_{n+1} + [(p - 1)/4]H_n \qquad (n \geq 1).$$

Note that, of course, the Lucas sequence corresponds to the case $p = 5$.

The following table, analogous to Whitford's gives the first ten terms of the sequences corresponding to the first five positive integers of the form $4k + 1$.

p	$\frac{p-1}{4}$	G_1	G_2	G_3	G_4	G_5	G_6	G_7	G_8	G_9	G_{10}
1	0	1	1	1	1	1	1	1	1	1	1
5	1	1	3	4	7	11	18	29	47	76	123
9	2	1	5	7	17	31	65	127	257	511	1025
13	3	1	7	10	31	61	154	337	799	1810	4207
17	4	1	9	13	49	101	297	701	1889	4693	12249

The following are some of the identities satisfied by the sequences H_n and G_n.

(1) $$\lim_{n\to\infty} \frac{H_{n+1}}{H_n} = (1 + \sqrt{p})/2,$$

(2) $$G_{2n} = G_n H_n,$$

(3) $$H_n^2 = H_{2n} + 2[(1 - p)/4]^n,$$

(4) $$H_n = G_{n+1} + [(p - 1)/4]G_{n-1},$$

(5) $$pG_n^2 = H_{2n} - 2[(1 - p)/4]^n.$$

The major change in the generalized identities occurs where $\alpha\beta = -1$ appears in the Fibonacci/Lucas identities, with $\alpha\beta = (1 - p)/4$ in their generalizations.

REFERENCES

1. A. K. Whitford. "Binet's Formula Generalized." *The Fibonacci Quarterly* 15 (1977):21.
2. V. E. Hoggatt, Jr. *Fibonacci and Lucas Numbers*. Boston: Houghton Mifflin, 1969.

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ON THE DISTRIBUTION OF QUADRATIC RESIDUES

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For p an odd prime, each of the integers 1, 2, ..., p - 1 is either a quadratic residue or a quadratic nonresidue. In [1], Andrews proves that the number of pairs of consecutive quadratic residues, the number of pairs of consecutive quadratic nonresidues, etc., are the values listed in Table 1. This note is a further investigation of the distribution of the quadratic residues and quadratic nonresidues which will include new proofs of the results in Table 1.

The integers 1, 2, ..., p - 1 can be partitioned into disjoint cells, in an alternate fashion, according to whether they are consecutive quadratic residues or quadratic nonresidues.