ON THE FORM OF PRIMITIVE FACTORS OF FIBONACCI NUMBERS

Brother U. Alfred

The factorization of members of the Fibonacci series : 1, 1, 2, 3, 5, 8, ... can be greatly facilitated if the form of the factors is known. Given a Fibonacci number F_n, if n is composite a first step may be taken by dividing out the value of all Fibonacci numbers F_m for which $m \mid n$. There remains a quotient whose factors are primes that have not yet appeared in the Fibonacci series. Following Jarden (Recurring Sequences, p. 8), let us call these primitive prime divisors.

If k(p) is the period for any such primitive prime divisor of F_n, it would follow that:

$$n \mid k(p)$$

If p is of the form $10x \pm 1$, $k(p) \mid p-1$, so that $n \mid p-1$. If p is of the form $10x \pm 3$, $k(p) \mid 2(p+1)$ so that $n \mid 2(p+1)$. Three cases will be distinguished: (A) n an odd quantity; (B) n of the form $2(2r+1)$; (C) n of the form

$$2^m(2r+1), \quad m \geq 2.$$

(A) <u>n odd</u>

For p of the form $10x \pm 1$, $p-1 = nk$ or $p = nk+1$. Since, however, n is odd, k would have to be even to give a prime, so that p would have to be of the form:

$$p = 2nk + 1, \quad (k = 1, 2, 3, ...)$$

If p is of the form $10x \pm 3$, $2(p+1) = kn$. Since n is odd, k is even. Letting $k = 2k'$,

$$p = k'n - 1$$

But again, k' will have to be even if p is to be odd. Thus combining the results for both cases, when n is odd the primitive factors of F_n are of the form:

$$p = 2kn \pm 1, \quad (k = 1, 2, 3, ...)$$

(B) $n = 2(2r+1)$

For p of the form $10x \pm 1$, $p-1 = 2k(2r+1)$

or $p = kn+1$, $(k = 1, 2, 3, \ldots\ldots)$

For $p = 10x \pm 3$, $2(p+1) = 2k(2r+1)$

or $p = k(2r+1) - 1$

To have a prime, k must be even, so that

$$p = nk - 1, \quad (k = 1, 2, 3, \ldots.)$$

Hence in case $n = 2(2r+1)$,

$$p = nk \pm 1, \quad (k = 1, 2, 3, \ldots.)$$

(C) $n = 2^m(2r+1)$, $m \geq 2$.

For p of the form $10x \pm 1$, $p-1 = 2^m k\,(2r+1)$

or $p = nk+1$, $(k = 1, 2, 3, \ldots\ldots)$

For p of the form $10x \pm 3$, $2(p+1) = 2^m k(2r+1)$

or $p = 2^{m-1}k(2r+1) - 1$

so that $p = \frac{nk}{2} - 1$, $(k = 1, 2, 3, \ldots\ldots)$

VERIFICATION

These forms were verified for all primitive prime factors found in Jarden's Tables (Recurring Sequences) up to $n = 100$. A sampling of the results is indicated in the following table where the notation at the right of the primitive prime factor gives the value of k and the sign of the form used.

n	FORM OF FACTORS	FACTORS
40	$40k \pm 1$, $20k - 1$	2161 (54, -)
41	$82k \mp 1$	2789 (34, +), 59369(724, +)
42	$42k \mp 1$	211(5, +)

n	FORM OF FACTORS	FACTORS
43	86k ± 1	433494437 (5040633, -)
44	44k ± 1, 22k-1	43(2, -), 307(14, -)
45	90k ± 1	109441 (1216, +)
46	46k ± 1	139(3, +), 461 (10, +)
47	94k ± 1	2971215073 (31608671, -)
48	48k ± 1, 24k-1	1103(46, -)
49	98k ± 1	97(1, -), 6168709 (62946, +)
50	50k ± 1	101(2, +), 151 (3, +)

****

FIBONACCI CENTURY MARK REACHED

It is with a sense of satisfaction that the editors of the Fibonacci Quarterly announce the complete factorization of the first hundred Fibonacci numbers. For the most part, this data is found in the volume, "Recurring Sequences", of D. Jarden. The finishing touches have been provided by John Brillhart of the University of San Francisco whose work was done on the computer at the University of California. This is just a small portion of his extensive factorizations of Fibonacci numbers, but it is a welcome contribution especially at this juncture.

The results are as follows: (* indicates a new factor)

F_{71} =(6673*)(46165371073*) (error in table)

F_{79} = (157)(92180471494753) (final factor a prime)

F_{83} = 99194853094755497 (a prime)

F_{89} = (1069)(1665088321800481) (final factor a prime)

F_{91} = (13^2) (233)(741469*) (159607993*)

F_{93} = (2) (557)(2417)(4531100550901)(final factor a prime)

F_{95} = (5) (37)(113)(761)(29641*)(67735001*) (error in table)

F_{97} = (193)(389)(3084989*)(361040209*)

****