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PROBLEM DEPARTMENT

P-1. Verify that the polynomials $\varphi_{k+1}(x)$ satisfy the differential equation

$$(1 + x^3)\, y'' + 3xy' - k(k+2)y = 0 \qquad (k = 0, 1, 2, \ldots)$$

P-2. Derive the series expansion

$$J_0(x) = \sum_{k=0}^{\infty} (-1)^k \left[I_k^2(\alpha) - I_{k+1}^2(\alpha) \right] F_{2k+1},$$

where J_0 and I_k are Bessel Functions.

P-3. Verify the reciprocal relation

$$x^n = (1/2^n) \sum_{r=0}^{[n/2]} (-1)^r \binom{n}{r} \frac{n-2r+1}{n-r+1} \varphi_{n+1-2r}(x), \quad n \geq 0.$$

P-4. Show that the determinant

$$\varphi_{k+1}(x) = \begin{vmatrix} 2x & i & 0 \ldots\ldots 0 & 0 \\ i & 2x & i \ldots\ldots 0 & 0 \\ 0 & i & 2x \ldots\ldots 0 & 0 \\ \cdot & \cdot & \cdots\cdots & \cdot \\ 0 & 0 & 0 \ldots\ldots 2x & i \\ 0 & 0 & 0 \ldots\ldots i & 2x \end{vmatrix} \qquad k \geq 1,$$

with $\varphi_0(x) = 0$, $\varphi_1(x) = 1$, and where $i = \sqrt{-1}$, satisfies the recurrence relation for $\varphi_k(x)$. Whence derive the expression

$$F_{k+1} = \varphi_{k+1}(1/2) = \begin{vmatrix} 1 & i & 0 \ldots 0 & 0 \\ i & 1 & i \ldots 0 & 0 \\ 0 & i & 1 \ldots 0 & 0 \\ \cdot & \cdot & \cdots\cdots & \cdot \\ 0 & 0 & 0 \ldots 1 & i \\ 0 & 0 & 0 \ldots i & 1 \end{vmatrix} \qquad k \geq 1$$

for the Fibonacci numbers.

P-5. Show that the Fibonacci polynomials may also be expressed by

$$\varphi_{k+1}(x) = \frac{2^k (k+1)!}{\sqrt{1+x^2}\,(2k+1)!} \frac{d^k}{dx^k} (1+x^2)^{k+1/2}, \quad (k \geq 0).$$