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For the inverse mapping $P \longrightarrow Z^n$ we need

$$f_0^{-1}(p_i) = \frac{(-1)^{\epsilon}(p_i - \epsilon)}{2},$$

where

$$\epsilon = \begin{cases} 0 & \text{for } p_i \text{ even,} \\ 1 & \text{for } p_i \text{ odd} \end{cases}.$$

Then

$$\begin{aligned} f_0^{-1} f_n^{-1}(p) &= f_0^{-1}(p_1, p_2, \cdots, p_n) \\ &= \left(f_0^{-1}(p_1), \cdots, f_0^{-1}(p_n)\right) \end{aligned}.$$

6. POLYNOMIAL COUNTING FUNCTIONS

It is quite easy to see from (1) that there are at least $n!$ polynomial counting functions of P^n (obtained by permuting $p_1, p_2, \cdots, p_n$). But for $n = 3$ besides these six polynomials of degree 3, there are six more polynomials of degree 4 obtained by composition of f_2 such as

$$f_2\left(f_2(p_1, p_2), p_3\right).$$

For $n = 4$ there are 360 polynomials, provided that different compositions yield distinct polynomials.

We are unable to determine the number of counting polynomials of P^n, except the case $n = 1$.

Theorem. The identical function $f_1(p_1) = p_1$ is the only polynomial mapping 1 - 1 from P onto itself.

Proof. Suppose g(p) is a counting polynomial of P. Consider the curve $y = g(x)$. It is clear that after a finite number of ups and downs the curve is monotone increasing (to $+\infty$). Let a be a positive integer such that (1) g(x) is monotone for $x \geq a$ and (2) $g(x) < g(a)$ for $x < a$. Since g(x) is a counting function of P, it has to satisfy

$$g(a) = a, \quad g(a + 1) = a + 1, \cdots .$$

For, if $g(a) < a$, then positive numbers $g(1), g(2), \cdots, g(a)$ cannot all be distinct, and if $g(a) > a$ then the curve must come down beyond a, contrary to (1). Now, by the Fundamental Theorem of Algebra we have $g(x) = x$ for all x.

Question. Are

$$x_1 + \binom{s_2 - 1}{2} \qquad \text{and} \qquad x_2 + \binom{s_2 - 1}{2}$$

the only two polynomials mapping 1 - 1 from P^2 onto P?

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