YE OLDE FIBONACCI CURIOSITY SHOPPE

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Let $S(X^2)_q$ symbolize the sum of the digits of X^2 on the base q. For example, $S(9^2)_5 = S(14^2)_5 = 5$ since $9_5^2 = 311$.

The following is a method for finding q such that $S(X^2)_q = X$ when X is given. For example $S(7^2)_8 = 7$ since $7_8^2 = 61$.

Step 1. List all the factors of X except X itself.

Step 2. List all the factors of X - 1.

Step 3. Multiply each factor of X by one of the factors of X - 1, discarding all products greater than X - 1. The retained products are the ten's digits of the X_q^2 that we seek.

Step 4. The unit's digits can be obtained by simple subtraction of the quantities in three from X.

Step 5. q can now be computed by simple arithmetic.

Example. $S(21^2)_q = 21$. Find all values of q.

Step I:			1	3	7	
Step II:	1	2	4	5	10	20
Step III:	1	2	4	5	10	20
	3	6	12	15		
	7	14				
Step IV:	1(20)	2(19)	4(17)	5(16)	10(11)	20(1)
	3(18)	6(15)	12(9)	15(6)	7(14)	14(7)

The quantities in parentheses are the unit's digits.

Step V: For example, for 5(16), $5b + 16 = 441$ in base ten so that $b = 85$ expressed as a base ten number. The bases taken in order are

421	211	106	85	43	22
141	71	36	29	61	31

The problem is: Why does this method work?.

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If eleven alternate terms of any Fibonacci sequence are added and divided by $L_{11}(199)$, the result is the middle term of the group of eleven terms added together.

Example. Using the series beginning 1, 4, $\cdots$,

157 + 411 + 1076 + 2817 + 7375 + 19308 + 50549 + 132339 + 346468 + 907065 + 2374727 = 3942292

Dividing by 199 gives 19308.

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