3. A.F. Horadam, "Generating Functions for Powers of a Certain Generalized Sequence of Numbers," *Duke Math. J.,* 32 (1965), pp. 437–446.
4. J.J. Kolodner, "On a Generating Function Associated with Generalized Fibonacci Sequences," *The Fibonacci Quarterly,* Vol. 3, No. 3 (Oct. 1965), pp. 272–278.
5. J. Riordan, "Generating Functions for Powers of Fibonacci Numbers," *Duke Math. J.,* 29(1962), pp. 5–12.

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A CONSTRUCTED SOLUTION OF $\sigma(n) = \sigma(n+1)$

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With $\sigma(n)$ the sum of the positive divisors of n, one finds that

$$\sigma(n) = \sigma(n+1) \tag{1}$$

for

$$n = 14, 206, \cdots, 18873, 19358, \cdots, 174717, \cdots. \tag{2}$$

Sierpinski [1] asked if (1) has infinitely many solutions. Earlier, Erdös had conjectured [2] that it does, but the answer is unknown. Makowski [3] listed the nine solutions of (1) with $n < 10^4$ and subsequently Hunsucker *et al* continued and found 113 solutions with $n < 10^7$. See [4] for a reference to this larger table.

It is unlikely that there are only finitely many solutions but, in any case, there is a much larger solution, namely,

$$n = 5559060136088313. \tag{3}$$

It is easily verified that the first, second, and fourth examples in (2) are given by

$$n = 2p, \quad n+1 = 3^m q, \tag{4}$$

where

$$q = 3^{m+1} - 4, \qquad p = (3^m q - 1)/2 \tag{4a}$$

are both prime, and m equals $1, 2,$ or 4. One finds that

$$\sigma(n) = \sigma(n+1) = \frac{1}{2}(9^{m+1} + 3) - 6 \cdot 3^m. \tag{4b}$$

The third and fifth examples in (2) are given by

$$n = 3^m q, \qquad n+1 = 2p \tag{5}$$

with the primes

$$q = 3^{m+1} - 10, \qquad p = (3^m q + 1)/2 \tag{5a}$$

for $m = 4$ and 5. Then

$$\sigma(n) = \sigma(n+1) = \frac{1}{2}(9^{m+1} + 9) - 15 \cdot 3^m. \tag{5b}$$

Our new solution (3) is given by $(5 - 5a)$ for $m = 16$. But there are no other examples of (5) or (4) for $m < 44$. While we do conjecture that there are infinitely many solutions of (1) we do not think that infinitely many solutions can be constructed in this way. D.H. and Emma Lehmer assisted us in these calculations.

REFERENCES

1. Waclaw Sierpiński, *Elementary Theory of Numbers,* Polska Akademia Nauk, Warsaw, Poland, 1964, p. 166.
2. Paul Erdös, "Some Remarks on Euler's ϕ Function and Some Related Problems," *Bull. AMS,* Vol. 51, 1945, pp. 540–544.
3. A. Makowski, "On Some Equations Involving Functions $\phi(n)$ and $\sigma(n)$," *Amer. Math. Monthly,* Vol. 67, 1960, pp. 668–670; "Correction," *ibid.,* Vol. 68, 1961, p. 650.
4. M. Lal, C. Eldridge, and P. Gillard, "Solutions of $\sigma(n) = \sigma(n+k)$," UMT 37, *Math. Comp.,* Vol. 27, 1973, p. 676.

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