form a complete system of orthogonal Latin squares for each fixed j, while for n odd we get a system of $(n-1)/2$ orthogonal Latin squares, each square occurring twice.

Theorem. If n is a power of 2 then there exist $n-1$ orthogonal Latin cubes of order n with the property that the corresponding plane sections form systems of $n-1$ orthogonal Latin squares.

If n is a power of an odd prime then there exist $n-1$ orthogonal Latin cubes with the property that the corresponding plane cross-sections in two directions form complete systems of orthogonal Latin squares, while the plane cross-sections in the third direction form a system of $(n-1)/2$ orthogonal Latin squares, each square occurring twice.

Finally we observe that if we have orthogonal k-cubes of orders m and n then we can form their Kronecker products to obtain orthogonal k-cubes of order mn. That is from orthogonal k-cubes

$$A^1 = (a^1_{i_1\cdots i_k}), \cdots, A^\ell = (a^\ell_{i_1\cdots i_k}); \qquad B^1 = (b^1_{i_1\cdots i_k}), \cdots, \qquad B^\ell = (b^\ell_{i_1\cdots i_k}),$$

where the a's run from 1 to m and the b's from 1 to n we can form the orthogonal k-cubes $C^1, \cdots, C^\ell$, where

$$C^j = (c^j_{i_1\cdots i_k}) \qquad \text{and} \qquad c^j_{i_1\cdots i_k} = (a^j_{i_1\cdots i_k}, b^j_{j_1\cdots j_k})$$

so that the c's run through all ordered pairs $(1,1), \cdots, (m,n)$ as the pairs $(i_1, j_1), \cdots, (i_k, j_k)$ run through these ordered pairs. Thus we have the following.

Corollary. If

$$n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_s^{\alpha_s} \qquad \text{and} \qquad q = \min_{1 \le j \le s} p_j^{\alpha_j}$$

then for any $k < q$ there exist at least $q-1$ orthogonal Latin k-cubes of order n.

The relation to finite k-dimensional projective spaces is not as immediate as it is for Latin squares, and we shall not discuss it here.

REFERENCES

1. J. Arkin, "A Solution to the Classical Problem of Finding Systems of Three Mutually Orthogonal Numbers in a Cube Formed by Three Superimposed $10 \times 10 \times 10$ Cubes," *The Fibonacci Quarterly*, Vol. 12, No. 2 (April 1974) pp. 133–140. Also, Sugaku Seminar, 13 (1974), pp. 90–94.
2. R.C. Bose, S.S. Shrikhande, and E.T. Parker, "Further Results on the Construction of Mutually Orthogonal Latin Squares and the Falsity of Euler's Conjecture," *Canadian J. Math.*, 12 (1960), pp. 189–203.
3. S. Chowla, P. Erdös and E.G. Straus, "On the Maximal Number of Pair-wise Orthogonal Latin Squares of a Given Order," *Canadian J. Math.*, 12 (1960), pp. 204–208.
4. J. Arkin and V.E. Hoggatt, Jr., "The Arkin-Hoggatt Game and the Solution of a Classical Problem," *Jour. Recreational Math.*, Vol. 6, No. 2 (Spring, 1973), pp. 120–122.

This research was supported in part by NSF Grant No. GP-28696.

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ON EXTENDING THE FIBONACCI NUMBERS TO THE NEGATIVE INTEGERS

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A sequence of positive integers defined by the formula

(1) $$x_{n+1} = ax_n + bx_{n-1}, \quad n \text{ a positive integer},$$

is said to be extendable to the negative integers if (1) holds for n any integer. See page 28 of [1]. The purpose of this note is to show that the Fibonacci numbers form a sequence which is extendable to the negative integers in a unique way. In this note only nontrivial integral sequences will be considered.

[Continued on Page 308.]