$$\binom{n-k}{a-k} = \binom{n-k}{n-a}, \quad \binom{2n}{2k} = \binom{2n}{2n-2k}, \quad \sum_{k=0}^{n} f(k) = \sum_{k=0}^{n} f(n-k).$$

Illustration. We show that (14) is equivalent to (10):

$$\sum_{k=0}^{a} \binom{2n+1}{2k}\binom{n-k}{a-k} = \sum_{k=0}^{a} \binom{2n+1}{2k}\binom{n-k}{n-a} = \sum_{k=0}^{n} \binom{2k+1}{2k}\binom{n-k}{n-a}$$

$$= \sum_{k=0}^{n} \binom{2n+1}{2n-2k}\binom{k}{n-a} = \sum_{k=0}^{n} \binom{2n+1}{2k+1}\binom{k}{n-a} .$$

Similarly (15) is equivalent to (8):

$$\sum_{k=0}^{a} \binom{2n}{2k+1}\binom{n-1-k}{a-k} = \sum_{k=0}^{a} \binom{2n}{2k+1}\binom{n-1-k}{n-1-a} = \sum_{k=0}^{n-1} \binom{2n}{2k+1}\binom{n-1-k}{n-1-a}$$

$$= \sum_{k=0}^{n-1} \binom{2n}{2n-2k-1}\binom{k}{n-1-a} = \sum_{k=0}^{n-1} \binom{2n}{2k+1}\binom{k}{n-1-a} .$$

The reader should now have no difficulty in showing that (16) is equivalent to (9), and that (17) is equivalent to (7). The equivalences are so complete and obvious that we wonder how anyone could miss them. Thus we have used the Moriarty formulas twice in our proofs of (3)–(6). Moriarty, Moriarty, all is Moriarty! "Indubitably my Dear Watson, indubitably."

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[Continued from Page 292.]

Theorem. The Fibonacci numbers form the only sequence of integers for which its extended sequence satisfies:

(i) $$x_{-n} = (-1)^{n+1}x_n, \quad n \text{ an integer},$$

(ii) any three consecutive terms of the sequence are relatively prime.

Proof. Let x_n be a sequence which satisfies (i) and (ii); then,

$$x_1 = ax_0 + bx_{-1} = ax_0 + bx_1 .$$

Hence,

(*) $$ax_0 = (1-b)x_1 .$$

Now,

$$x_0 = ax_{-1} + bx_{-2} = ax_1 - b(ax_1 + bx_0),$$

which implies that

$$(1+b^2)x_0 = ax_1(1-b) = a^2x_0 ,$$

using (*). Since the sequence is nontrivial x_0 and x_1 cannot both be 0. If $x_0 \neq 0$; then $a^2 = 1+b^2$, which implies that $a = \pm 1$ and $b = 0$. In either of these cases, (ii) will not hold. Hence, $x_0 = 0$. From (*) it follows that $b = 1$.

Thus far, the sequence hs the form $x_0 = 0, x_1, ax_1, \cdots$; hence, in order to satisfy (ii), x_1 must equal 1. This yields a sequence of the form

$$x_0 = 0, 1, a, a^2+1, a^3+2a, \cdots$$

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