A RECIPROCAL SERIES OF FIBONACCI NUMBERS

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Theorem

$$\frac{1}{F_1}+\frac{1}{F_2}+\frac{1}{F_4}+\frac{1}{F_8}+\frac{1}{F_{16}}+\cdots=\frac{7-\sqrt{5}}{2}\,.$$

Proof

$$\frac{1}{F_1}+\frac{1}{F_2}+\frac{1}{F_4}+\cdots+\frac{1}{F_{2^n}}=3-F_{2^n-1}/F_{2^n}$$

is easily proved by induction using Binet's formula, and the theorem follows by letting $n \to \infty$. The result resembles the formula

$$\sqrt{m}=\frac{(m-1)\alpha_n}{4\beta_{n-1}}-\frac{m-1}{2}\left(\frac{1}{\beta_n}+\frac{1}{\beta_{n+1}}+\cdots\right)\,,$$

where

$$m>1,\ \alpha_1=2\,\frac{m+1}{m-1},\ \alpha_{n+1}=\alpha_n^2-2,\ \beta_0=1,\ \beta_n=\alpha_1\alpha_2\cdots\alpha_n\,.$$

(Reference 1.

Some curious properties of Fibonacci numbers appeared in [2]; for example,

$$\Delta_{48}^2 5^{F_n}=5^{F_{n+96}}-2\cdot 5^{F_{n+48}}+5^{F_n}$$

is a multiple of $2^{12}3^5 7^3=341,397,504$ for $n=1,2,3,\cdots$.

REFERENCES

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2. I.J. Good and R.A. Gaskins, "Some Relationships Satisfied by Additive and Multiplicative Congruential Sequences, with Implications for Pseudo-random Number Generation," *Computers in Number Theory: Proceedings of the Science Research Council Atlas Symposium No. 2* at Oxford, 18–23 August 1969 (Academic Press, Aug. 1971, eds. A.O.L. Atkin and B.J. Birch), 125–136.

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