A SPECIAL CASE OF THE GENERALIZED FIBONACCI SEQUENCE OVER AN ARBITRARY RING WITH IDENTITY

M. W. BUNDER

The University of Wollongong, Wollongong, N.S.W., Australia

DeCarli [1] introduced the sequence $\{M_n\}$ of elements of an arbitrary ring with identity S by

$$M_{n+2} = A_1M_{n+1} + A_0M_n \quad \text{for} \quad n \geq 0,$$

where M_0, M_1, A_0 and A_1 are arbitrary elements of S. He considers in particular the case which he calls the sequence $\{F_n\}$ with

$$F_{n+2} = A_1F_{n+1} + A_0F_n \quad \text{for} \quad n \geq 0,$$

where $F_0 = 0$ (the zero of the ring) $F_1 = I$ (the identity) and A_0 and A_1 are arbitrary elements of S.

A number of DeCarli's theorems can be simplified in the special case where $A_0A_1 = A_1A_0$. We use the following theorems which are easily proved by induction.

Theorem 1. $\quad A_0F_n = F_nA_0, \quad A_1F_n = F_nA_1 \quad$ for all n.

Theorem 2. $\quad F_nF_m = F_mF_n \quad$ for all m and n.

Thus DeCarli's Theorem 3

$$F_nF_{n+r} - F_{n+r}F_n = F_nF_rA_0F_{n-1} - F_{n-1}A_0F_1F_n$$

becomes trivial.

Also we can prove that F_n commutes with any element of S which commutes with A_0 and A_1. In particular when A_0 and A_1 commute with all elements of S so does F_n.

The two parts of DeCarli's Corollary 1 can thus be rewritten as

$$F_{n+1}F_{n-1} - F_n^2 = A_0(F_{n-1}^2 - F_nF_{n-2}).$$

In the same way as above for the general sequence, if M_0, M_1, A_0 and A_1 all commute with each other then all Mn's commute with each other and with A_0 and A_1.

REFERENCE

1. D.J. DeCarli, "A Generalized Fibonacci Sequence Over an Arbitrary Ring," *The Fibonacci Quarterly*, Vol. 8, No. 2 (March 1970), pp. 182–184.

****