

6. J.W. Moon, *Counting Labeled Trees*, Canadian Mathematical Congress, Montreal, 1970.
 7. G. Prins, "On the Automorphism Group of a Tree, Doctoral Dissertation, University of Michigan, 1957.

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[Continued from Page 278.]

$$U(2n+1) = 2T_n + p(n).$$

Secondly, if one places all the partition summands in a line separated by plusses, then one deletes the plus signs at the end of each partition, so that

$$P(n) = U(n) + S(n) - p(n),$$

leading to

$$P(2n) = U(2n) + S(2n) - p(2n) = 2T_n + T_n - p(2n) = 3T_n - n - 1, \quad n \geq 1.$$

Equivalently,

$$\begin{aligned} P(2n+2) &= 3T_{n+1} - (n+1) - 1 = \frac{3(n+1)(n+2)}{2} - n - 2 \\ &= \frac{3(n+1)n}{2} + \frac{3(n+1)2 - 2(n+2)}{2} \\ &= 3T_n + 2n + 1, \quad n \geq 0. \end{aligned}$$

More easily, we have

$$P(2n+1) = U(2n+1) + S(2n+1) - p(2n+1) = 2T_n + p(2n+1) + T_n - p(2n+1) = 3T_n,$$

which finishes the proof.

We note that the generating function for each sequence given is easily written since the triangular numbers are involved, as

$$\sum_{n=0}^{\infty} P(2n+1)x^n = \frac{3}{(1-x)^3}$$

$$\sum_{n=0}^{\infty} P(2n+2)x^n = \frac{4-x^2}{(1-x)^3}$$

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