INTERESTING PROPERTIES OF LAGUERRE POLYNOMIALS

N. C. Mohanty
Systems Control, Inc., Palo Alto, California 94304

Recent interest in optical communication has added to the importance of study of Laguerre polynomials [1] and distribution. We will establish two propositions which arise in studies of Laguerre distribution [2].

Definition.

$$L_n^\alpha(R) \triangleq \sum \binom{n+\alpha}{n-i} \frac{(-1)^i}{i!} R^i \,,$$

where

$$R^i \triangleq \int x^i p(x)dx \,.$$

Proposition 1:

$$\int L_n^\alpha(x)p(x)dx = L_n^\alpha(R) \,.$$

Proof.

$$\int L_n^\alpha(x)p(x)dx = \int \sum_{i=0}^{n} \binom{n+\alpha}{n-i} \frac{(-1)^i}{i!} x^i p(x)dx = \sum_{i=0}^{w} \binom{n+\alpha}{n-i} \frac{(-1)^i}{i!} \int x^i p(x)dx$$

$$= \sum_{i=0}^{n} \binom{n+\alpha}{n-i} \frac{(-1)^i}{i!} R^i = L_n^\alpha(R).$$

Proposition 2. If $R^{i+j} = R^i R^j$, then

$$\int L_n^\alpha(x) L_m^\beta(x)p(x)dx = L_n^\alpha(R) L_m^\beta(R).$$

Proof.

$$\int L_n^\alpha(x) L_m^\beta(x)p(x)dx = \sum \sum \binom{m+\beta}{m-j}\binom{n+\alpha}{n-i} \frac{(-1)^{i+j}}{i!\,j!} \int x^{i+j} p(x)dx$$

$$= \sum_{i=0}^{n} \sum_{j=0}^{m} \binom{n+\alpha}{n-i}\binom{m+\beta}{m-j} \frac{(-1)^{i+j}}{i!\,j!} R^{i+j}$$

$$= \left\{ \sum \binom{n+\alpha}{n-i} \frac{(-1)^i}{i!} R^i \right\} \left\{ \sum \binom{m+\beta}{m-j} \frac{(-1)^j}{j!} R^j \right\}$$

$$= L_n^\alpha(R) L_m^\beta(R) \,.$$

CONCLUSION

It is interesting to note that if $p(x) > 0$ and $\int p(x)dx = 1$ and $R^i < \infty \ \forall i$, then R^i are called moments of the random variable x. Expectation of Laguerre polynomials of random variables is Laguerre polynomials of moments.

REFERENCES

1. Sansone, G., *Orthogonal Functions,* Interscience, New York, 1959.
2. Gagliardi, R. M., "Photon Counting and Laguerre Detection," *IEEE Transactions in Information Theory,* January 1972, Vol. IT-8, No. 1.

****