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Let q^b denote one of the $p_i^{a_i}$ and P denote $q^{b-2}(q-1)^2$. Now,

$$q^{b-2}(q-1)^2 = q^{b-1}(q-2+1/q). \tag{3}$$

From (3), it can be seen that $P > 1$, for all q, and that $P > 8$, for all $q \geqslant 11$. Furthermore, for $q < 11$, the following table can be obtained, by checking the right side of (3) for the case $b = 1$, and the left side of (3) for the case $b \geqslant 2$.

Prime q	3	3	5	5	7	7
Exponent b	2	3	1	2	1	2
P greater than or equal to	4	8	2	8	4	8

Hence, (2) holds for $p - 1$ possibly equal to $2\cdot3$, $2\cdot3^2$, $2\cdot5$, $2\cdot7$, $2\cdot3\cdot5$, $2\cdot3\cdot7$ ($a = 1$); $4\cdot3$, $4\cdot5$, $4\cdot3\cdot5$ ($a = 2$); or $8\cdot3$ ($a = 3$); and (2) fails to hold for all other choices. These combinations lead to the primes 7, 11, 13, 19, 31, 43, 61.

Theorem 3. If p is a prime greater than 5, then the primitive roots are not consecutive.

Proof. For the primes excluded in the Lemma, the primitive roots are: for 7 – 3, 5; for 11 – 2, 6, 7, 8; for 13 – 2, 6, 7, 11; for 19 – 2, 3, 10, 13, 14, 15; for 31 – 3, 11, 12, 13, 17, 21, 22, 24; for 43 – 3, 5, 12, 18, 19, 20, 26, 28, 29, 30, 33, 34; for 61 – 2, 6, 7, 10, 17, 18, 26, 30, 31, 35, 43, 44, 51, 54, 55, 59. None of these primes have consecutive primitive roots.

Now, let p denote a prime for which the Lemma applies and suppose that k is a positive integer for which $k^2 \leqslant p - 1$. Then,

$$k^2 - (k-1)^2 = 2\cdot k - 1 < 2\cdot k \leqslant 2\sqrt{p-1} \leqslant \phi(p-1).$$

Therefore, consecutive squares appear within a span less than $\phi(p-1)$. Since squares are quadratic residues, and therefore not primitive roots, no string of consecutive primitive roots can be of length $\phi(p-1)$. Consequently, the primitive roots are not consecutive.

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