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ADDITIVE PARTITIONS I

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David Silverman in July 1976 found the following property of the Fibonacci Numbers. This Theorem I was subsequently proved by Ron Evans, Harry L. Nelson, David Silverman, and Krishnaswami Alladi with myself, all independently.

Theorem I. The Fibonacci Numbers uniquely split the positive integers, N, into two sets A_0 and A_1 such that

$$A_0 \cup A_1 = N$$
$$A_0 \cap A_1 = \phi$$

and so that no two members of A_0 nor two members of A_1 add up to a Fibonacci number.

Theorem. (Hoggatt) Every positive integer $n \neq F_k$ is the sum of two members of A_0 or the sum of two members of A_1.

Theorem. (Hoggatt) Using the basic ideas above the Fibonacci Numbers uniquely split the Fibonacci Numbers, the Lucas Numbers uniquely split the Lucas Numbers and uniquely split the Fibonacci Numbers, and $\{5F\}_{n=2}^{\infty}$ uniquely splits the Lucas Sequence.

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