$$\frac{A(nk+u)}{B(nk+u)} = \frac{\alpha_1 + \beta_1(m_2/m_1)^n}{\alpha_2 + \beta_2(m_2/m_1)^n}\,.$$

Hence limit of

$$c = \lim_{n\to\infty} \{A(nk+u)/B(nk+u)\} = \alpha_1/\alpha_2\,.$$

Notice that α_1 and α_2 are quadratic irrationals. Is the limit unique? Yes, by Theorem 3, we have

$$\begin{vmatrix} A(nk+u) & B(nk+u) \\ A(nk+v) & B(nk+v) \end{vmatrix} = \det(L^n) \begin{vmatrix} A(u) & B(u) \\ A(v) & B(v) \end{vmatrix} = \pm\sigma,$$

σ is a constant. Then

$$\frac{A(nk+u)}{B(nk+u)} - \frac{A(nk+v)}{B(nk+v)} = \frac{\pm\sigma}{B(nk+u)B(nk+v)}$$

As $n \to \infty$,

$$\frac{A(nk+u)}{B(nk+u)} - \frac{A(nk+v)}{B(nk+v)} = 0\,.$$

If $c = [a_1, \cdots, a_j, \overline{a_{j+1}, \cdots, a_{j+k}}]$, then take

$$P = \{a_1, \cdots, a_j, \overline{a_{j+1}, \cdots, a_{j+k}}\}$$

as the generating sequence, the limit of c is then given by

$$\lim_{n\to\infty} \frac{A(nk+u+j)}{B(nk+u+j)}, \quad u > 0.$$

Remark. Actually we have proved just now a theorem in continued fractions: A continued fraction c is peridic iff α is a quadratic irrational, for which c is the continued fraction expansion.

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ADDITIVE PARTITIONS II

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Theorem (Hoggatt). The Tribonacci Numbers,

$$1, 2, 4, 7, 13, 24, \cdots, T_{n+3} = T_{n+2} + T_{n+1} + T_n\,,$$

with 3 added to the set uniquely *split* the positive integers and each positive integer $n \ne 3$ or $\ne T_m$ is the sum of two elements of A_0 or two elements of A_1. (See "Additive Partitions I," page 166.)

Conjecture. Let A split the positive integers into two sets A_0 and A_1 and be such that $p \notin A \cup \{1,2\}$, and p is representable as the sum of two elements of A_0 or the sum of two elements of A_1. We call such a set *saturated* (that is $A \cup \{1, 2\}$). Krishnaswami Alladi asks: "Does a *saturated* set imply a unique *additive* partition?" My conjecture is that the set $\{1, 2, 3, 4, 8, 13, 24, \cdots\}$ is *saturated* but does not cause a unique split of the positive integers. Here we have added 3 and 8 to the Tribonacci sequence and deleted the 7. Paul Bruckman points out that this fails for 41. EDITOR

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