$$M(n,k)/2^{n-1} = M(n-1,k)/2^{n-2} + kM(n-2,k)/2^{n-3}.$$

As $M(1,k) = 1$ and $M(2,k) = 2$ one can use induction to prove that $M(n,k)$ is divisible by 2^{n-1}.

Also solved by David G. Beverage, Wray G. Brady, Paul S. Bruckman, Herta T. Freitag, David Zeitlin, and the Proposer.

OPERATIONAL IDENTITY

B-339 Proposed by Gregory Wulczyn, Bucknell University, Lewisburg, Pennsylvania.

Establish the validity of E. Cesàro's symbolic Fibonacci-Lucas identity $(2u+1)^n = u^{3n}$; after the binomial expansion has been performed, the powers of u are used as either Fibonacci or Lucas subscripts. (For example, when $n = 2$ one has both $4F_2 + 4F_1 + F_0 = F_6$ and $4L_2 + 4L_1 + L_0 = L_6$.)

Solution by Graham Lord, Université Laval, Québec, Canada.

For a fixed K, since both

$$F_K a + F_{K-1} = a^k \quad \text{and} \quad F_K b + F_{K-1} = b^K,$$

the n^{th} power of each when added (algebraically) will give the result

$$(F_K u + F_{K-1})^n = u^{Kn}.$$

The desired equation is the special case when $K = 3$.

Also solved by David G. Beverage, Wray G. Brady, Paul S. Bruckman, Herta T. Freitag, Ralph Garfield, H. Turner Laquer, A. G. Shannon, David Zeitlin, and the Proposer.

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[Continued from page 284.]

Solution by David Beverage, San Diego Community College, San Diego, California.

By using the polynomials $P_{2n+1}(x)$* expressed explicitly as

(1) $$P_{2n+1}(x) = \sum_{r=0}^{n} 5^{n-r}(-1)^{kr} \frac{(2n+1)[(2n-r)!]}{r!\,(2n+1-2r)!} x^{2n+1-2r} \;**$$

and selecting $m = 2n+1$, obtain

(2) $$Q = \frac{F_{mp}}{F_p} = F_p \cdot H \pm m,$$

where H is a polynomial in F_p.

Clearly,

$$(F_p, m) \mid (F_p, Q).$$

Select $m > 1$ with integral coefficients and $m \mid F_p$ $(m \not\equiv 0\ (p))$ in order that $(F_p, Q) > 1 \cdots$. The above conditions are satisfied for many numbers m and p. One example: $p = 7$ and $m = 13$ produces

$$\frac{F_{91}}{F_7} = 358465123875040793 = Q \qquad \text{and} \qquad (F_7, Q) = 13 > 1.$$

Many other interesting divisor relationships may be obtained from the polynomials $P_{2n+1}(x)$.

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*David G. Beverage, "A Polynomial Representation of Fibonacci Numbers," *The Fibonacci Quarterly*, Vol. 9 No. 5 (Dec. 1971)

**David G. Beverage, "Polynomials $P_{2n+1}(x)$ Satisfying $P_{2n+1}(F_k) = F_{(2n+1)k}$," *The Fibonacci Quarterly*, Vol. 14, No. 3 (Oct. 1976), pp. 197–200.

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