$$\sum_{k=0}^{4} p(4,k) = 816 = 24.34 = 24F_9 .$$

Similarly

$$\sum_{k=0}^{4} q(4,k) = 1320 = 24.35 = 24F_{10} .$$

It is clear that

(4.6) $$p(n,n) = q(n,n) = 1 \qquad (n = 0, 1, 2, \cdots).$$

Taking $k = n - 1$ in (4.3) and $n$ in (4.4), we get

(4.7) $$q(n, n-1) = p(n, n-1) + n \qquad (n \geqslant 1)$$

and

(4.8) $$p(n+1, n) = 2(n+1) + S(n, n-1) - 1,$$

respectively. Since

$$S(n, n-1) = \tfrac{1}{2}n(n-1),$$

it follows that

(4.9) $$\begin{cases} p(n, n-1) = \tfrac{1}{2}n(n+1) \\ q(n, n-1) = \tfrac{1}{2}n(n+3) = p(n+1, n) - 1 . \end{cases}$$

As for $k = 0$, it is evident from (3.4) that

$$\lim_{u=0} F_{u+n} = F_n ,$$

so that

(4.10) $$p(n,0) = n!\, F_{2n}, \qquad q(n,0) = n!\, F_{2n+1} .$$

It would be of interest to find combinatorial interpretations of $p(n,k)$ and $q(n,k)$.

## REFERENCES


1. M. W. Bunder, "On Halsey's Fibonacci Function," *The Fibonacci Quarterly,* Vol. 13, No. 2 (April 1975), pp. 209–210.
2. E. Halsey, "The Fibonacci Number $F_u$ where $u$ is not an Integer," *The Fibonacci Quarterly,* Vol. 3, No. 2 (April 1965), pp. 147–152.


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[Continued from p. 245.]

(As a corollary, note that we have proved

$$F_{m+1}F_{m-1} - F_m^2 = \det(g^m) = (-1)^m .)$$

Then the lemma implies there is a sequence $\{m_j\}$ for which

$$g^{m_j} \to 1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

in the $p$-adic topology. Thus we can choose $\{m_j\}$ so that $d(1, g^{m_j}) < p^{-j}$. Then $p^j$ divides $F_{m_j}$ and $1 - F_{m_j+1}$, which proves the theorem.

It is clear that one can vary $G$ and $g$ in the argument above to prove a class of theorems related to the well known one quoted.

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