the three results occurring at the end of Section 3, it is possible to deduce the coefficient of x^{k-1} when k is even. The coefficient is

$$\frac{k}{k-1}\left\{1 + \frac{3(k-2)}{2(k-3)} + \frac{3 \cdot 5(k-2)(k-4)}{2 \cdot 4(k-3)(k-5)} + \frac{3 \cdot 5 \cdot 7(k-2)(k-4)(k-6)}{2 \cdot 4 \cdot 6(k-3)(k-5)(k-7)} + \cdots\right\},$$

the expression within the brackets terminating, since k is even.

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FIBONACCI RATIO IN A THERMODYNAMICAL CASE

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Consider the thermodynamics of an infinite chain of alternately spaced $2N$ molecules of donors and acceptors ($N \to \infty$), and assume there is an average of one mobile electron per molecule (as is quite common for some one-dimensional organic crystals [1, 2]).

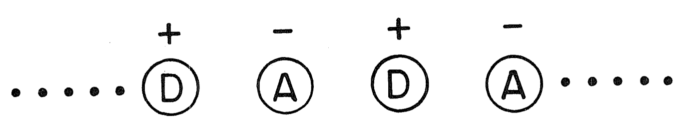

FIGURE 1

Each molecule may contain a maximum of two such electrons and as the temperature is raised two electrons may jump onto the same molecule. Because electrons repel each other, it costs an energy U_D or U_A to put two electrons on a molecule type D or type A, respectively; a common situation is that

$$U_D >> U_A.$$

Under these conditions, it can cost almost no energy to have sites A doubly occupied, while double occupancy of sites D is effectively eliminated.

In the grand-canonical ensemble, the partition function Z of the electrons can then be approximated by

(1) $$Z = \lambda^N,$$

where $\lambda = z_A z_D$; z_A and z_D being the partition functions "per molecule" of type A and D, respectively. In terms of the fugacity [3], z and λ can be obtained easily, in fact,

(2) $$z_A \cong 1 + 2z + z^2 = (1 + z)^2$$

and

(3) $$z_D \cong 1 + 2z.$$

The three terms in (2) (in ascending powers of z) correspond to zero occupancy, single occupancy (with spin up or down), and double occupancy (respectively) of sites A. In (3) there is no z^2 term, because double occupancy of sites D is effectively eliminated.

In the grand-canonical ensemble, the positive quantity z is determined [3] by fixing the "average" number of particles (in this case, electrons). Since we have an average of one electron per site, z will be determined by the condition [3]

(4) $$z\frac{\partial\lambda}{\partial z} = 2\lambda.$$

Substituting for λ in terms of (2) and (3) and simplifying, (4) gives the cubic equation

(5) $$(z + 1)(z^2 - z - 1) = 0$$

for z. Finally, the positive

(6) $$z_+ = \frac{1 + \sqrt{5}}{2}$$

The Fibonacci ratio is the only appropriate physical solution of (5) for the fugacity z. From the grand-partition function Z and the numerical value of λ,

(7) $$\lambda = (1 + z)^2(1 + 2z) = z_+^7,$$

the thermodynamics [3] then easily follows.

In particular, the entropy S that arises from the number of possible arrangements of the electrons in the chain is given by

(8) $$\frac{S}{k_B} = 5N \ln z ,$$

where k_B is Boltzmann's constant.

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ON GROUPS GENERATED BY THE SQUARES

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1. INTRODUCTION

It was known that the quaternion group and the octic group could not be generated by the squares of any group [5, pp. 193-194]. A natural question is which groups are generated by the squares of some groups. Clearly, groups of odd order and simple groups are generated by their own squares. In this paper, we show in a concrete manner that abelian groups are generated by the squares of some groups, and we show that every group is contained in the set of squares of some group. We give conditions for the dihedral and dicyclic groups to be generated by the squares of some groups. Also we show that several classes of nonabelian 2-groups cannot be generated by the squares of any group.

2. NOTATIONS AND DEFINITIONS

Throughout this paper, all groups considered are assumed to be finite. For a group G, we let G^2 denote the set of squares, $I(G)$ the group of innerautomorphisms, $A(G)$ the group of automorphisms, $Z(G)$ the center, $|G|$ the order of G, G^1 the commutator subgroup. For any subset S of G, $\langle S\rangle$ denotes the subgroup generated by S. G is called an <u>S-group</u> if it is generated by the squares of some group L; to be more precise, there is a group L such that $\langle L^2\rangle$ is isomorphic to G.

3. CLASSES OF S-GROUPS

In a group of odd order, every element is a square; therefore, it is an S-group. A simple group is also an S-group since it is generated by its own squares; for, if the set of squares generates a proper subgroup, it would be a normal subgroup with abelian quotient. We next show that an abelian group is an S-group.