Theorem 5.5: Let G be a nonabelian 2-group with

$$\langle G^2 \rangle = \langle a \rangle \times \langle b \rangle, \text{ where } |a| = {}^{n}, \ |b| = 2.$$

Suppose $\langle G^2 \rangle$ contains exactly one element c which is not a square; also suppose that either $c \notin G'$ or $|G'| > 2$, and $[G{:}G'] = 4$. G is not an S-group.

The proof of this theorem is similar to that for Theorem 5.4. An example is the group G of order 3_2 with presentation

$$a^4 = b^2 = c^2 = d^2 = 1, \ d^{-1}ad = a,$$

$$d^{-1}cd = cb, \ c^{-1}ac = a^{-1},$$

where a^2 and b are central elements. Here

$$G' = \langle G^2 \rangle = \langle a^2, b \rangle,$$

and the element $a^2 b$ is not a square. By Theorem 5.5 G is not an S-group.

REFERENCES

1. W. Burnside. *Theory of Groups of Finite Order*. Dover, 1955.
2. W. Burnside. "On Some Properties of Groups Whose Orders are Powers of Primes." *Proc. Lond. Math. Soc.* (2), 11 (1912):225-245.
3. R. D. Carmichael. *Introduction to the Theory of Groups of Finite Order*. Dover, 1956.
4. G. A. Miller. *Collected Works*, V. University of Illinois, 1959.
5. W. R. Scott. *Group Theory*. Prentice-Hall, 1964.
6. H. S. Sun. "Embedding a Set into a Group." Unpublished doctoral dissertation, University of New Brunswick, 1969.
7. H. S. Sun. "A Group Whose Squares Generate a Dicyclic Group." *American Math. Monthly* 78, No. 9 (1971).
8. H. S. Sun. "Embedding a Semigroup in a Ring." *The Fibonacci Quarterly* 13, No. 1 (1975):50.
9. H. S. Sun. "Embedding a Group in the *p*th Powers." *The Fibonacci Quarterly* 16, No. 1 (1978):4.

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A PRIMER ON STERN'S DIATOMIC SEQUENCE—II

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PART II: SPECIAL PROPERTIES

In 1929, D. H. Lehmer, at Brown University, presented a summary [1] of discovered results concerning Stern's sequence. Also, in July 1967, some additional results were reported by D. A. Lind [2]. In order to standardize the results, we will define Stern's sequence to be $s(i,j)$ where

(1) $s(i,0) = 1$, for $i = 0, 1, 2, \ldots$
(2) $s(0,j) = 0$, for $j = 1, 2, 3, \ldots$

(3) $s(n,2k) = s(n,k)$, for $n,k = 1, 2, 3, \ldots$
(4) $s(n,2k+1) = s(n-1,k) + s(n-1,k+1)$.

A table follows:

STERN NUMBER TABLE

	Column																		
Row	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	3	2	3	1	2	1	1	0	0	0	0	0	0	0	0	0	0	0
4	1	4	2	5	1	5	3	4	1	3	2	3	1	2	1	1	0	0	0
5	1	5	4	7	3	8	5	7	2	7	5	8	3	7	4	5	1	4	3
6	1	6	5	9	4	11	7	10	3	11	8	13	5	12	7	9	2	9	7
7	1	7	6	11	5	14	9	13	4	15	11	18	7	17	10	13	3	14	11
8	1	8	7	13	6	17	11	16	5	19	14	23	9	22	13	17	4	19	15
9	1	9	8	15	7	20	13	19	6	23	17	28	11	27	16	21	5	24	19
10	1	10	9	17	8	23	15	22	7	27	20	33	13	32	19	25	6	29	23
11	1	11	10	19	9	26	17	25	8	31	23	38	15	37	22	29	7	34	27
12	1	12	11	21	10	29	19	28	9	35	26	43	17	42	25	33	8	39	31
13	1	13	12	23	11	32	21	31	10	39	29	48	19	47	28	37	9	44	35
14	1	14	13	25	12	35	23	34	11	43	32	53	21	52	31	41	10	49	39
15	1	15	14	27	13	38	25	37	12	47	35	58	23	57	34	45	11	54	43
16	1	16	15	29	14	41	27	40	13	51	38	63	25	62	37	49	12	59	47
17	1	17	16	31	15	44	29	43	14	55	41	68	27	67	40	53	13	64	51
18	1	18	17	33	16	47	31	46	15	59	44	73	29	72	43	57	14	69	55

The authors will attempt to move quickly through the properties of these numbers without proof.

(1) The number of terms in row n is $2^n + 1$.
(2) The sum of all terms in row n is $3^n + 1$.
(3) The average value of all terms approaches $(3/2)^n$.
(4) The table is symmetric:
$s(n,k) = s(n, 2^n + 2 - k)$ for $2^n + 2 - k \geq 0$.
(5) In three successive terms a, b, c, $(a+c)/b$ is an integer.
(See Part I [3], Sections 4 and 11.)
(6) Given a, b, and c again, then b occurs at
$s(n-k, (a+c-b)/2b)$. (See [3], Section 4.)
(7) Any two consecutive terms are relatively prime.
(See [3], Section 5.)
(8) Any ordered pair can only appear once in the table.
(See [3], Section 6.)
(9) If $a/b = (k, k_1, k_2, \ldots, k_m, r_{m-1})$, then a and b appear together in line $(k + k_1 + k_2 + \cdots + k_m + r_{m-1} - 1)$.
(See [3], Section 10.)
(10) The number of times that an element k can appear in the row $k-1$, and all succeeding rows, is Euler's function $0(k)$.

(11) "p" is a prime if and only if it appears exactly $(p - 1)$ times in line $(p - 1)$.

(12) $s(n,r)$ will appear again at locations $s(n + k,\ 2^k(r - 1) + 1)$ for $k = 1, 2, 3, \ldots$.

(13) If the sequence r_1, r_2 occurs in row n, $r_1 > r_2$, the smallest element in row $n + k$ positioned between r_1 and r_2 is
$s(n + k,\ 2^k r) = r_1 + kr_2$.

(14) In any row, there are two equal terms greater than all others in the row.

(15) For Fibonacci followers:
$s(n,r) = F_{n+1}$, for $r = (2^{n-1} + 2 + \{1 + (-1)^n\})/3 - 1$,
and it is the largest element in the row.
(See [3], p. 65; notation changed to standard form.)

Not all of the discovered results are considered here, since there are remote connections to so many areas of number theory.

REFERENCES

1. D. H. Lehmer. "On Stern's Diatomic Series." *American Math. Monthly* 36 (1929):59-67.
2. D. A. Lind. "An Extension of Stern's Diatomic Series." *Duke Math. J.*, July 7, 1967.
3. Christine & Robert Giuli. "A Primer on Stern's Diatomic Sequence, Part I." *The Fibonacci Quarterly* 17, No. 2 (1979):103-108.

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SUMS OF PRODUCTS: AN EXTENSION

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The purpose of this note is to extend the results of Berzsenyi [1] and Zeilberger [3] on sums of products by using the generalized sequence

$$\{W_n(a,b;p,q)\}$$

described by the author in [2], the notation of which will be assumed.

Equation (4.18) of [2, p. 173] tells us that

$$W_{n-r}W_{n+r+t} - W_nW_{n+t} = eq^{n-r}U_{r-1}U_{r+t-1}. \tag{1}$$

Putting $n - r = k$ and summing appropriately, we obtain

$$\sum_{k=0}^{n} W_kW_{k+2r+t} = \sum_{k=0}^{n} W_{k+r}W_{k+r+t} + eU_{r-1}U_{r+t-1}\sum_{k=0}^{n} q^k. \tag{2}$$

Values $t = 1$, $t = 0$ give, respectively,

$$\sum_{k=0}^{n} W_kW_{k+2r+1} = \sum_{k=0}^{n} W_{k+r}W_{k+r+1} + eU_{r-1}U_r\sum_{k=0}^{n} q^k, \tag{3}$$

and