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ADDENDA TO "PYTHAGOREAN TRIPLES CONTAINING
FIBONACCI NUMBERS: SOLUTIONS FOR $F_n^2 \pm F_k^2 = K^2$ "

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In a recent correspondence from J. H. E. Cohn, it was learned that Ljunggren [1] has proved that the only square Pell numbers are 0, 1, and 169. (This appears as an unsolved problem, H-146, in [2] and as Conjecture 2.3 in [3].) Also, if the Fibonacci polynomials $\{F_n(x)\}$ are defined by

$$F_0(x) = 0, F_1(x) = 1, \text{ and } F_{n+2}(x) = xF_{n+1}(x) + F_n(x),$$

then the Fibonacci numbers are given by $F_n = F_n(1)$, and the Pell numbers are $P_n = F_n(2)$. Cohn [4] has proved that the only perfect squares among the sequences $\{F_n(a)\}$, a odd, are 0 and 1, and whenever $a = k^2$, a itself. Certain cases are known for a even [5].

The cited results of Cohn and Ljunggren mean that Conjectures 2.3, 3.2, and 4.2 of [3] are true, and that the earlier results can be strengthened as follows.

If $(n, k) = 1$, there are no solutions in positive integers for

$$F_n^2(a) + F_k^2(a) = K^2, \quad n > k > 0, \text{ when } a \text{ is odd and } a \geq 3.$$

This is the same as stating that no two members of $\{F_n(a)\}$ can occur as the lengths of legs in a primitive Pythagorean triangle, for a odd and $a \geq 3$.

When $a = 1$, for Fibonacci numbers, if

$$F_n^2 + F_k^2 = K^2, \quad n > k > 0,$$

then $(n, k) = 2$, and it is conjectured that there is no solution in positive integers. When $a = 2$, for Pell numbers, $P_n^2 + P_k^2 = K^2$ has the unique solution $n = 4$, $k = 3$, giving the primitive Pythagorean triple 5-12-13.

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