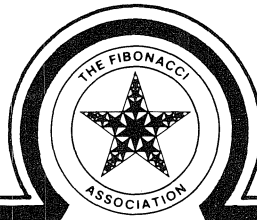


# The Fibonacci Quarterly

THE OFFICIAL JOURNAL OF THE FIBONACCI ASSOCIATION

VOLUME 21  
NUMBER 2



MAY  
1983

## CONTENTS

A Note on the Fibonacci Sequence of Order $K$ and the Multinomial Coefficients .....	<i>Andreas N. Philippou</i>	82
Associated Hyperbolic and Fibonacci Identities .....	<i>E. Ehrhart</i>	87
Sequences Generated by Self-Replicating Systems .....	<i>Wesley A. Darbro &amp; Georg von Tiesenhausen</i>	97
Why Are 8:18 and 10:09 Such Pleasant Times? .....	<i>M.G. Monzingo</i>	107
On Exponential Series Expansions and Convolutions .....	<i>M.E. Cohen &amp; D.L. Hudson</i>	111
Notes on Fibonacci Trees and Their Optimality .....	<i>Yasuichi Horibe</i>	118
A Note on Fibonacci Cubature .....	<i>Robert D. Girse</i>	129
A Family of Polynomials and Powers of the Secant .....	<i>O.R. Ainsworth &amp; J. Neggers</i>	132
A Variant of Nim and a Function Defined by Fibonacci Representation .....	<i>David R. Hale</i>	139
Kronecker's Theorem and Rational Approximation of Algebraic Numbers .....	<i>Wayne M. Lawton</i>	143
Elementary Problems and Solutions .....	<i>Edited by A.P. Hillman</i>	147
Advanced Problems and Solutions .....	<i>Edited by Raymond E. Whitney</i>	153

## PURPOSE

The primary function of **THE FIBONACCI QUARTERLY** is to serve as a focal point for widespread interest in the Fibonacci and related numbers, especially with respect to new results, research proposals, challenging problems, and innovative proofs of old ideas.

## EDITORIAL POLICY

**THE FIBONACCI QUARTERLY** seeks articles that are intelligible yet stimulating to its readers, most of whom are university teachers and students. These articles should be lively and well motivated, with new ideas that develop enthusiasm for number sequences or the exploration of number facts. Illustrations and tables should be wisely used to clarify the ideas of the manuscript. Unanswered questions are encouraged, and a complete list of references is absolutely necessary.

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All back issues of **THE FIBONACCI QUARTERLY** are available in microfilm or hard copy format from **UNIVERSITY MICROFILMS INTERNATIONAL, 300 NORTH ZEEB ROAD, DEPT P.R., ANN ARBOR, MI 48106.**

1983 by

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# *The Fibonacci Quarterly*

Founded in 1963 by Verner E. Hoggatt, Jr. (1921-1980)  
Br. Alfred Brousseau, and I.D. Ruggles

*THE OFFICIAL JOURNAL OF THE FIBONACCI ASSOCIATION  
DEVOTED TO THE STUDY  
OF INTEGERS WITH SPECIAL PROPERTIES*

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