## SIDNEY'S SERIES

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(Submitted October 1984)

Sidney's series of numbers may qualify in two ways for being considered a part of the world of pure mathematics. This series is, as far as this author knows, without practical application, and is very beautiful. The series was discovered by the author's daughter Sidney Larison in 1968 when she was about age fifteen.

Using two-digit numerals, five series can be produced (or six if you count zero). Using three-digit numerals, nineteen series can be produced (or twenty if you count zero). Using four-digit numerals, eleven series can be produced (or twelve if you count zero).

To produce the series using two-digit numerals, start with any two-digit numeral, for example,
23.

Add them together and affix their sum, as, 235.

Add the last two digits together and affix their sum, as, 2358.

Add the last two digits together and affix their sum, modulo 10, always dropping from the sum the digit in tens place if there is one, as,

23583, and then, 235831... .
Continue the process until the first two digits repeat.
The first series in the set is now complete.
To produce the second series in the set of six, start with any two-digit numeral not included in the first series and repeat the process.

To produce the third, fourth, and fifth series in the set, select any as-yet-unused two-digit numeral and repeat the process.

The sixth series in the set simply contains zero.
These six series of numbers contain all of the two-digit numerals from 00 through 99 and none will appear more than once. Each two-digit numeral can fit one series and no other.

Series utilizing numerals of three, four, or any desired number of digits may be produced. To produce the set of twenty series using three-digit numerals, select any three-digit numeral, add the digits and affix their sum, modulo 10 , as, $12361078503 \ldots$.

When the first three digits repeat, that series in the set is complete.
The twenty series in the set using three-digit numerals utilize every numeral from 000 through 999 and none is used more than once. Each three-digit numeral appears in one series and no other.

Completing the set based on four-digit numerals proved to be too large a task to be accomplished by hand so the computer was used. William G. Sjostrom of Modesto, California, wrote in BASIC the necessary programs to write the set of four-digit series. There turn out to be only twelve series in the set-six sets of 1560 digits each, two sets of 312 digits each, three sets of 5 digits each, and zero.

When the six series of numbers based on two-digit numerals are equally spaced in a set of six concentric circles, some interesting properties become apparent. Any series which contains more than one zero will contain four of them and they will be equally spaced around the circle. Pairs of digits which are directly opposite each other in the circle will add up to either zero or ten.

No attempt has as yet been made to place the ten thousand digits of the four-digit series in a set of twelve concentric circles, but an inspection of the lists shows that those series containing 000 more than once will contain it four times and they will be equally spaced around the circle. As in the series based on two-digit numerals, single digits directly opposite each other in the circle will have as their sum either zero or ten.

The twenty series of numbers based on three-digit numerals when equally spaced in a set of twenty concentric circles exhibit no interesting properties in relation to zero. Nor do digits directly opposite each other in the circle add up to ten or zero. However, a study of this series in a search for interesting properties revealed a fascinating property shared by all series so far tested.

To examine this property, proceed as follows:
List, horizontally, a string of digits as they occur in any series from any set, as, from the set based on three digits,
6095487940...

Under it write another series from the same set, as,
6095487940...
2035869380...

Add, modulo 10.
Your result, in this case $8020246220 . .$. , will follow all the rules for producing a series from that number of digits and will, indeed, be another series from that set!

It works without fail! Add together, in order, the digits from two or more series from the same set and the result will be a series in the same set!

Multiply, modulo 10, in order, the digits from any series by the same numeral, and your result will be a series in the same set.

For example, take
$6095487940 \ldots$ from the three-digit set.
Multiply by $\underline{3} \underline{3} \underline{3} \underline{3} \underline{3} \underline{3} \underline{3} \underline{3} \underline{3} \underline{3} \ldots$ Your result, in this case
$8075241720 \ldots$ follows all the rules and is a member of the three-digit set.

There may be other interesting properties to be discovered in these series of numbers. No one knows, for example, how many series will be required to complete the set based on five-digit numerals or what properties they will display.

The author predicts that the set based on five-digit numerals will display the same properties as the other sets in relation to addition and multiplication, and forty series will be required to complete the set.


THE SET OF SERIES BASED ON TWO-DIGIT NUMERALS

